PHYS 942 homework assignment #05

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PHYS 942
November 13, 2019

Names (≤3, write clearly): ______________________________

Due: Friday, November 22, at the lecture. **Show all your steps!**

1. (*30 points*) Covariant Ohm's law: In the rest frame of a conducting medium the current density satisfies Ohm's law $\mathbf{J}' = \sigma\mathbf{E}'$, where σ is the conductivity and the primes denote quantities in the rest frame.

   (a) Taking into account the possibility of a convection current (i.e., a finite charge density ρ') as well as conduction current, show that the covariant generalization of Ohm's law is:

   $$J^\alpha - \frac{1}{c^2}(U_\beta J^\beta)U^\alpha = \frac{\sigma}{c}F^{\alpha\beta}U_\beta$$

   where U^α is the 4-velocity of the medium.

   (b) Show that if the medium has a velocity $\mathbf{v} = c\beta$ with respect to some inertial frame that the 3-vector current in that frame is

   $$\mathbf{J} = \gamma\sigma[\mathbf{E} + \beta\times\mathbf{B} - \beta(\beta\cdot\mathbf{E})] + \rho\mathbf{v}$$

   where ρ is the charge density observed in that frame.

2. (*30 points*) A particle of mass M decays at rest into a number of smaller particles such that the sum of the masses of the decay products is less by an amount of ΔM.

   (a) Show that the maximum kinetic energy of the ith particle with mass m_i is

   $$(T_i)_{\max} = \Delta M\Big(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\Big)$$

   (b) determine the maximum kinetic energies in MeV and the ratios to ΔM for each of the following decay events (find the respective rest masses in the literature):

   $$\mu \to e + \nu + \tilde{\nu}$$
   $$K^+ \to \pi^+ + \pi^- + \pi^-$$
   $$K^\pm \to \mu^\pm + \pi^0 + \nu$$

3. (*30 points*) The universe is permeated by a "sea" of blackbody radiation at a temperature of 2.7 K. Cosmic γ-ray photons will collide with these background photons and can create e^+ - e^- pairs if sufficient energy is available in the head-on collision "rest frame."

   (a) Calculate the minimum γ-ray energy to create an e^+ - e^- pair. Such collisions explain why there is an apparent upper energy limit for cosmic rays.

   (b) There is also some evidence for an x-ray background with photon energies of about 1 keV. Repeat the calculation for such a "background x-ray photon sea."

4. (*30 points*) Lorentz transformation of acceleration in inertial systems: A coordinate system K' moves with a velocity $\mathbf{v}$ relative to another system K. In K' a particle has a velocity $\mathbf{u}'$ and an acceleration $\mathbf{a}'$. Use the Lorentz transformation to show that in the system K the component of acceleration parallel to $\mathbf{v}$ is:

$$\mathbf{a}_{\parallel} = \frac{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}{\left(1 + \frac{\mathbf{v}\cdot\mathbf{u}'}{c^2}\right)^3}\mathbf{a}'_{\parallel}$$

5. (*30 points*) Trappist-1d is an exoplanet orbiting the nearby star Trappist and is considered to be possibly habitable for humans. It lies 39.5 light years away from Earth. On 1/1/2025 you decide that life on this Earth has become too miserable, and embark on a trip to Trappist-1d (financed by Musk and Trump). Conveniently, your spacecraft accelerates at 14 m/s^2 to allow you to sit rather comfortably in your recliner and still complete the trip in a timely manner. You accelerate to the half-way point and then decelerate at the same rate to come to a stop at Trappist-1d. You take a good look, and you immediately decide that this planet is no better than Earth. You return in the same fashion. When you return:

   (a) How much older are you according to your own clock?

   (b) What date is it on Earth now?

   (c) What was the largest speed you ever attained, in terms of km/s, β, and γ, as measured by the earthlings you left behind?

   (d) How would (a), (b), and (c) have turned out if you accelerated at only 0.2 m/s^2?

   (e) When your spaceship touches down back on Earth nobody lives here any more. In the mean time humanity found a better planet, and they all took off. They sent you a SMS while you were in transit at the common cell-phone frequency of 1900 MHz, but your swear you never got it. What went wrong?

   Obviously, you need the result from the previous problem to solve this.