PHYS 942 homework assignment #02

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PHYS 942
September 25, 2019

Names ($\leq$3, write clearly): ______________________________

Due: Friday, October 04, at the lecture. **Show all your steps!**

1. (*10 points*) Show that for one of the cases of reflection, the amplitude of the reflected wave vanishes for the incidence angle i_B if:

$$i_B = \tan^{-1}\left(\frac{n'}{n}\right).$$

   The angle i_B is called the *Brewster angle*.

2. (*20 points*) The properties of the ionosphere with respect to EM wave propagation is well described by the plasma dispersion in the high frequency limit $\epsilon(\omega)/\epsilon_0 = 1 - \omega_p^2/\omega^2$, where ω_p is the plasma frequency.

   Consider an ionosphere layer where ω_p jumps suddenly from zero to a given value at a height h.

   (a) For waves with polarization both perpendicular to the plane of incidence and parallel to the plane of incidence, show from the Fresnel equations that for $\omega > \omega_p$ there is a range of angles of incidence for which reflection is not total, but for larger angles there is total reflection back towards earth.

   (b) A radio amateur operating at a wavelength of 30 meters during the day finds that she can receive distant stations located more than 1000 km away, but none closer. Assuming that the signals are being reflected at the ionosphere F layer at 300 km height, calculate the electron density.

   (c) Explain why late at night she can no longer receive any station more than 1000 km away.

   (d) Explain why AM stations (0.5 to 1.5 MHz) can be received closeby (<100 km) but typically not farther away.

   (e) Explain why FM stations (80 to 110 MHz) can be received closeby (<100 km) but typically not farther away.

   (f) Explain why short wave (SW) stations can be received halfway around the globe, say from 10,000 km away.

   (g) The Russian Sputnik I satellite transmitted its (in)famous "beep – beep – beep" at frequencies of 20 and 40 MHz. Explain why was this a useful scientific experiment. And why did the beeps have an enourmous propaganda value?

3. (*50 points*) EM wave penetration: An EM wave of frequency ω is incident normally on a flat surface of an non-permeable excellent conductor ($\mu = \mu_0$, $\epsilon = \epsilon_0$, and $\sigma \gg \omega\epsilon_0$), which fills the region $z{>}0$. Calculate:

   (a) The electric field amplitude and phase of the reflected wave.

   (b) The time-averaged Poynting flux into the conductor.

4. (*20 points*) Consider a circularly polarized plane wave propagating in a homogeneous medium in the z-direction. The wave has a *finite extent* in x and y given by $E_0(x, y)$, such that the lateral extent of the wave is large compared to its wavelength and that $E_0(x, y)$ is slowly varying. Such a wave could be produced, for example, by a laserpointer.

   Show that

   (a) the electric field of the wave is given by

   $$\mathbf{E}(x, y, z, t) = \Big[E_0(x, y)(\mathbf{e}_x \pm i\mathbf{e}_y) + \frac{i}{k}(\frac{\partial E_0}{\partial x} \pm i\frac{\partial E_0}{\partial x})\mathbf{e}_z\Big]e^{ikz-i\omega t}.$$

   (b) the magnetic field is approximately given by

   $$\mathbf{B} \simeq \pm i\sqrt{\mu\epsilon}\,\mathbf{E}.$$