PHYS 942 homework assignment #01

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PHYS 942
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Names (≤3, write clearly): ______________________________

Due: Wednesday, September 18, at the lecture. **Show all your steps!**

1. (*20 points*) The "Northern Pass Project" is a (controversial) proposal to build a power line from the Canadian border to southern NH in order to supply cheap Canadian hydro-electric power to New England. The line would be 192 miles long with a capacity of 1090 MW delivered power. For simplicity assume DC running over 12 aluminum wires of 25 mm (1 inch) diameter.

   (a) How large are the transmission losses depending on the line voltage, with typical voltages of 55 kV, 110 kV, 220 kV, 380 kV, 500 kV, and 765 kV? Create a plot of power loss versus line voltage for voltages from 50 kV to 1 MV.

   (b) Assuming that building the line costs $500M plus $1M per kV line voltage (because higher voltage means higher towers, longer insulators, bigger transformers), that each MWh lost costs $80, and that financing costs are 12% of the capital. Assume the financing is done with a balloon loan, i.e., no amortization and the loan is paid back in full at the end of the term. For a life time of 10 years, what is the most economical voltage? Plot cost versus voltage.

   (c) How much Al is needed?

   (d) Just considering the most economical voltage, would it make financial sense to replace the aluminum ($2/kg) with the same mass of copper ($8/kg)?

2. (*10 points*) Show that $\mathbf{S}_1 = \Phi\mathbf{J}$ instead of the "classical" Poynting vector in the quasi-static limit (i.e., when the displacement current can be neglected, corresponding to $L/c \ll T$) also satisfies Poynting's theorem. Here, Φ is the electric potential and $\mathbf{J}$ is the current density.

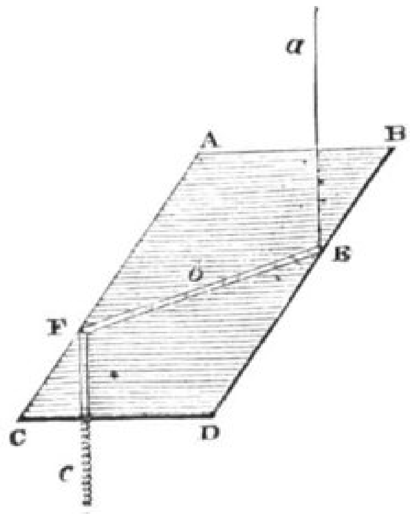
3. (*10 points*) Consider an infinite wire of radius a running along the z-axis and carrying a current I.

   (a) Calculate the Poynting vector inside and outside the wire and show that the energy flow into the wire equals the Ohmic dissipation in the wire.

   (b) Repeat the calculation for $\mathbf{S}_1$ defined above. What is the difference?

   (c) Sketch $\mathbf{S}$ and $\mathbf{S}_1$.

4. (*40 points*) The figure below shows a *Fresnel rhombus*. Incident light coming from a is internally relected twice and exits at c. For a suitable choice of parameters, a Fresnel rhombus converts linearly polarized light into circularly polarized light. Hint: Jackson mentions the *Fresnel rhombus* on page 308.



(a) Explain how the Fresnel rhombus works.

(b) Determine the orientation of the polarization vector of the incoming light and the angle of incidence at E that is required to convert plane polarized light into circularly polarized light if the index of refraction of the glass is n=1.51.

Hint: consider the phase change of the light by the total internal reflection.

5. (*20 points*) Consider an isothermal atmosphere with temperature T in equilibrium.

(a) Using the momentum equation for hydrostatics $\nabla p = n\mathbf{g}$, where $\mathbf{g}$ is the gravitational acceleration and p the atmospheric pressure, calculate the density $n(z)$ as a function of altitude z (positive upward) if the density on the ground is n_0.

(b) Now assume radiation is entering the atmosphere from above at an angle ϑ to the zenith and at radiation density J_∞. Assuming the cross section of absorption is σ show that the intensity of the radiation obeys:

$$dJ(z) = n(z)\sigma \frac{dz}{\cos\vartheta} J(z)$$

whereas the absorption rate per unit volume is $q(z) = \cos\vartheta(dJ/dz)$. Now, show that

$$q(z) = q_0 \exp(1 - y - \frac{1}{\cos\vartheta} \exp(-y))$$

where $y = (z - z_0)/H$, H is the scale height from part (a), q_0 is the maximum of $q(z)$ over z, and z_0 is the height at which the maximum occurs, i.e., $q_0 = q(z_0)$.

(c) Make plots of $q(z)/q_0$ for ϑ = 0° , 45° , and 85° . You should see that the absorption rate (in case of the ionosphere at which electrons are produced, but this formalism is also valid for other photo-reactions such as the ozone layer) forms a relatively thin layer. Such a layer is called a *Chapman layer*.