PHYS 942 homework assignment #07

Department of Physics
University of New Hampshire
Prof. J. Raeder, J.Raeder@unh.edu

PHYS 942
November 26, 2018

Names ($\leq$3, write clearly): _____

Due: Wednesday, December 5, 2018, at the lecture. **Show all your steps!**

1. (*40 points*) Covariant Ohm's law: In the rest frame of a conducting medium the current density satisfies Ohm's law $\mathbf{J}' = \sigma\mathbf{E}'$, where σ is the conductivity and the primes denote quantities in the rest frame.

   (a) Taking into account the possibility of a convection current (i.e., a finite charge density ρ') as well as conduction current, show that the covariant generalization of Ohm's law is:

   $$J^\alpha - \frac{1}{c^2}(U_\beta J^\beta)U^\alpha = \frac{\sigma}{c}F^{\alpha\beta}U_\beta$$

   where U^α is the 4-velocity of the medium.

   (b) Show that if the medium has a velocity $\mathbf{v} = c\beta$ with respect to some inertial frame that the 3-vector current in that frame is

   $$\mathbf{J} = \gamma\sigma[\mathbf{E} + \beta\times\mathbf{B} - \beta(\beta\cdot\mathbf{E})] + \rho\mathbf{v}$$

   where ρ is the charge density observed in that frame.

2. (*30 points*) A particle of mass M decays at rest into a number of smaller particles such that the sum of the masses of the decay products is less by an amount of ΔM.

   (a) Show that the maximum kinetic energy of the ith particle with mass m_i is

   $$(T_i)_{\max} = \Delta M\Big(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\Big)$$

   (b) determine the maximum kinetic energies in MeV and the ratios to ΔM for each of the following decay events (find the respective rest masses in the literature):

   $$\mu \to e + \nu + \tilde{\nu}$$
   $$K^+ \to \pi^+ + \pi^- + \pi^-$$
   $$K^\pm \to \mu^\pm + \pi^0 + \nu$$

3. (*30 points*) The universe is permeated by a "sea" of blackbody radiation at a temperature of 2.7 K. Cosmic γ-ray photons will collide with these background photons and can create e^+ - e^- pairs if sufficient energy is available in the head-on collision "rest frame."

   (a) Calculate the minimum γ-ray energy to create an e^+ - e^- pair. Such collisions explain why there is an apparent upper energy limit for cosmic rays.

   (b) There is also some evidence for an x-ray background with photon energies of about 1 keV. Repeat the calculation for such a "background x-ray photon sea."