PHYS 942 homework assignment #06

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PHYS 942
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Names ($\leq$3, write clearly): ______________________________

Due: Monday, November 19, 2018, at the lecture. **Show all your steps!**

1. (*20 points*) Show that two consecutive Lorentz transformations in the same direction with velocity v_1 and v_2, respectively, are equivalent to a single transformation with $v = (v_1 + v_2)/(1 + v_1 v_2/c^2)$. Show this in two ways:

   (a) By using the explicit addition formula.

   (b) By using the boost parameter and the 4x4 L.T. transformation matrix.

2. (*10 points*) A plane monochromatic electromagnetic wave propagating in free space is incident normal to a plane mirror surface where it is reflected. Obtain the frequency of the reflected wave in the case that the mirror moves at speed v, not necessarily small compared to c, with respect to the observer. Assume that $\mathbf{v}$, $\mathbf{k}$, and $\mathbf{n}$ (the normal to the mirror surface) are all parallel.

3. (*30 points*) Seeing versus observing:

   (a) In the following, assume β=0.9. A rod of length L'=10 m is aligned with, and moving along the x-axis towards you at speed v. You, the observer, are located slightly above the x-axis so you can see both ends of the rod (Note: *seeing* means that a photon arrives at your eye.)

   (i) What is the rod's length L that you *observe*? How would you measure it?

   (ii) What is the rod's length L_s^+ that you *see*? Why is it different from (i)? (The effect is called *abberation*).

   (iii) What length L_s^- would you *see* if the rod moves away from you at speed v?

   (iv) How would (ii) and (iii) turn out if Galilean relativity applied?

4. (*30 points*) Lorentz transformation of acceleration in inertial systems: A coordinate system K' moves with a velocity $\mathbf{v}$ relative to another system K. In K' a particle has a velocity $\mathbf{u}'$ and an acceleration $\mathbf{a}'$. Use the Lorentz transformation to show that in the system K the components of acceleration parallel and perpendicular to $\mathbf{v}$ are:

$$\mathbf{a}_{\parallel} = \frac{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}{\left(1 + \frac{\mathbf{v}\cdot\mathbf{u}'}{c^2}\right)^3}\mathbf{a}'_{\parallel}$$

$$\mathbf{a}_{\perp} = \frac{\left(1 - \frac{v^2}{c^2}\right)}{\left(1 + \frac{\mathbf{v}\cdot\mathbf{u}'}{c^2}\right)^3}\left(\mathbf{a}'_{\perp} + \frac{\mathbf{v}}{c^2}\times(\mathbf{a}'\times\mathbf{u}')\right)$$

5. (*30 points*) Proxima Centauri-b is an exoplanet orbiting the nearby star Proxima Centauri and considered to be possibly habitable for humans. It lies 4.2 light years away from Earth. On 1/1/2025 you decide that life on this Earth has become too miserable, and embark on a trip to Proxima Centauri-b (financed by Musk and Trump). Conveniently, your spacecraft accelerates at 12 m/s^2 to allow you to sit rather comfortably in your recliner and still complete the trip in a timely manner. You accelerate to the half-way point and then decelerate at the same rate to come to a stop at Proxima Centauri-b. You take a good look, and you immediately decide that this planet is no better than Earth. You return in the same fashion. When you return:

   (a) How much older are you according to your own clock?

   (b) What date is it on Earth now?

   (c) What was the largest speed you ever attained, in terms of km/s, β, and γ, as measured by the earthlings you left behind?

   (d) How would (a), (b), and (c) have turned out if you accelerated at only 0.2 m/s^2?

   (e) When your spaceship touches down back on Earth nobody lives here any more. In the mean time humanity found a better planet, and they all took off. They sent you a SMS while you were in transit at the common cell-phone frequency of 1900 MHz, but your swear you never got it. What went wrong?

   Obviously, you need the result from the previous problem to solve this.