# PHYS 942 homework assignment #04

Department of Physics
University of New Hampshire
Prof. J. Raeder, J.Raeder@unh.edu

PHYS 942
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Names ($\leq$3, write clearly): ______________________________

Due: Monday, October 22, at the lecture. **Show all your steps!**

1. (*40 points*) A transmission line (basically a coaxial cable) consisting of two concentric circular cylinders of metal with conductivity $\sigma$ is filled with lossless dielectric $(\mu, \epsilon)$. The inner cylinder has an outer radius $a$, and the outer cylinder has an inner radius $b$. A TEM wave is propagated along the line.

   (a) Calculate the time-averaged power flow along the line when $H_0$ is the peak azimuthal magnetic field at the surface of the inner conductor.

   (b) Calculate the attenuation constant $\gamma$ such that $P(z) = P_0 \exp(-2\gamma z)$ along the line.

2. (*40 points*) Transverse electric and magnetic waves are propagated along a hollow, right circular cylinder with inner radius $R$ and conductivity $\sigma$.

   (a) Find the cutoff frequencies of the various TE and TM modes. Determine numerically the lowest cutoff frequency in terms of the tube radius and the ratio of cutoff frequencies of the next 3 higher modes to that of the dominant mode. For this part assume that the conductivity of the cylinder is infinite.

   (b) Calculate the attenuation constantsof the waveguide as a function of frequency for the lowest two distinct modes (TE or TM) and plot them as a function of frequency.

   The following relations will come in handy:

   $$\int_0^a J_n(x_{lm}\rho/a) J_n(x_{ln}\rho/a)\rho d\rho = \frac{1}{2}a^2 J_{l+1}(x_{mn})^2,$$

   where $x_{mn}$ is the $n^{th}$ root of $J_m$, and:

   $$J_{m+1}(z) = \frac{m}{z}J_m(z) - J'_m(z),$$

   which implies:

   $$J_{m+1}(x_{mn}) = -J'_m(x_{mn}).$$

3. (*20 points*) A waveguide is constructed so that the cross section of the guide forms a right triangle with the sides of length $a$, $a$, $\sqrt{2}a$. The medium inside has $\mu_r$=1 and $\epsilon_r$=1. Assuming infinite conductivity for the walls, determine the possible modes of propagation and their cutoff frequencies.