PHYS 942 homework assignment #02

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PHYS 942
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Names ($\leq$3, write clearly): ______________________________

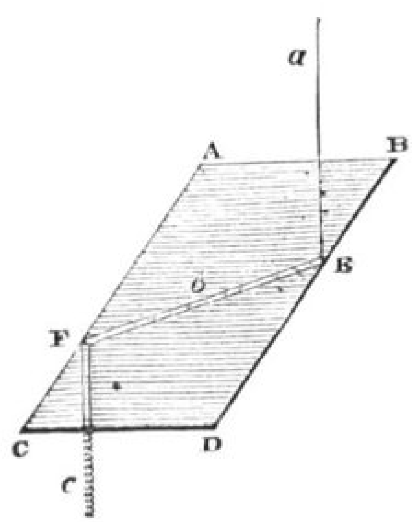
Due: Friday, October 1, at the lecture. **Show all your steps!**

1. (*10 points*) Show that for one of the cases of reflection, the amplitude of the reflected wave vanishes for the incidence angle i_B if:

$$i_B = \tan^{-1}\left(\frac{n'}{n}\right).$$

The angle i_B is called the *Brewster angle*.

2. (*40 points*) The figure below shows a *Fresnel rhombus*. Incident light coming from a is internally relected twice and exits at c. For a suitable choice of parameters, a Fresnel rhombus converts linearly polarized light into circularly polarized light. Hint: Jackson mentions the *Fresnel rhombus* on page 308.



(a) Explain how the Fresnel rhombus works.

(b) Determine the orientation of the polarization vector of the incoming light and the angle of incidence at E that is required to convert plane polarized light into circularly polarized light if the index of refraction of the glass is n=1.51.

Hint: consider the phase change of the light by the total internal reflection.

3. (*40 points*) Two plane semi-infinite slabs of the same uniform, isotropic, nonpermeable, lossless dielectric with index of refraction n are parallel and separated by an air gap (n=1) of width d. A plane electromagnetic wave of frequency ω is incident on the gap from one of the slabs with with angle of incidence i. For linear polarization *both* parallel to *and* perpendicular to the plane of incidence,

   (a) make a sketch of the problem and determine which wave fields you need to consider.

   (b) calculate the ratio of power transmitted into the second slab to the incident power and the ratio of refelected to incident power.

   (c) for i greater than the critical angle for total internal reflection, plot the ratio of transmitted power to incident power as a function of d measured in units of wavelength in the gap.

   The effect studied here is the basis of a *Fabry-Perot* interferometer.

4. (*20 points*) Consider an isothermal atmosphere with temperature T in equilibrium.

   (a) Using the momentum equation for hydrostatics $\nabla p = n\text{g}$, where g is the gravitational acceleration and p the atmospheric pressure, calculate the density $n(z)$ as a function of altitude z (positive upward) if the density on the ground is n_0.

   (b) Now assume radiation is entering the atmosphere from above at an angle ϑ to the zenith and at radiation density J_∞. Assuming the cross section of absorption is σ show that the intensity of the radiation obeys:

$$dJ(z) = n(z)\sigma \frac{dz}{\cos\vartheta} J(z)$$

   whereas the absorption rate per unit volume is $q(z) = \cos\vartheta(dJ/dz)$. Now, show that

$$q(z) = q_0 \exp(1 - y - \frac{1}{\cos\vartheta}\exp(-y))$$

   where $y = (z - z_0)/H$, H is the scale height from part (a), q_0 is the maximum of $q(z)$ over z, and z_0 is the height at which the maximum occurs, i.e., $q_0 = q(z_0)$.

   (c) Make plots of $q(z)/q_0$ for ϑ = 0° , 45° , and 85° . You should see that the absorption rate (in case of the ionosphere at which electrons are produced, but this formalism is also valid for other photo-reactions such as the ozone layer) forms a relatively thin layer. Such a layer is called a *Chapman layer*.

5. (*20 points*) EM wave penetration: An EM wave of frequency ω is incident normally on a flat surface of an non-permeable excellent conductor ($\mu = \mu_0$, $\epsilon = \epsilon_0$, and $\sigma \gg \omega\epsilon_0$), which fills the region $z>0$. Calculate:

   (a) The electric field amplitude and phase of the reflected wave.

   (b) The time-averaged Poynting flux into the conductor.