PHYS 942 homework assignment #01

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PHYS 942
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Names ($\leq$3, write clearly): ______________________________

Due: Friday, September 14, at the lecture. **Show all your steps!**

1. (*20 points*) The "Northern Pass Project" is a (controversial) proposal to build a power line from the Canadian border to southern NH in order to supply cheap Canadian hydroelectric power to New England. The line would be 192 miles long with a capacity of 1090 MW delivered power. For simplicity assume DC running over 12 aluminum wires of 25 mm (1 inch) diameter.

   (a) How large are the transmission losses depending on the line voltage, with typical voltages of 55 kV, 110 kV, 220 kV, 380 kV, 500 kV, and 765 kV? Create a plot of power loss versus line voltage for voltages from 50 kV to 1 MV.

   (b) Assuming that building the line costs \$500M plus \$1M per kV line voltage (because higher voltage means higher towers, longer insulators, bigger transformers), that each MWh lost costs \$80, and that financing costs are 12% of the capital. Assume the financing is done with a balloon loan, i.e., no amortization and the loan is paid back in full at the end of the term. For a life time of 10 years, what is the most economical voltage? Plot cost versus voltage.

   (c) How much Al is needed?

   (d) Just considering the most economical voltage, would it make financial sense to replace the aluminum (\$2/kg) with the same mass of copper (\$8/kg)?

2. (*20 points*) Show that $\mathbf{S}_1 = \Phi\mathbf{J}$ instead of the "classical" Poynting vector in the quasi-static limit (i.e., when the displacement current can be neglected, corresponding to $L/c \ll T$) also satisfies Poynting's theorem. Here, Φ is the electric potential and $\mathbf{J}$ is the current density.

3. (*20 points*) Consider an infinite wire of radius a running along the z-axis and carrying a current I.

   (a) Calculate the Poynting vector inside and outside the wire and show that the energy flow into the wire equals the Ohmic dissipation in the wire.

   (b) Repeat the calculation for $\mathbf{S}_1$ defined above. What is the difference?

(c) Sketch $\mathbf{S}$ and $\mathbf{S}_1$.

4. (*40 points*) Assume periodic fields such as $\mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x})e^{-i\omega t}$, where $\mathbf{E}(\mathbf{x})$ is in general a complex vector. Calculate the time-averaged

$$\frac{1}{2}\int_V \mathbf{J}^* \cdot \mathbf{E}\, d^3x$$

using Maxwell's equations and express the result in terms of $\mathbf{S} = \frac{1}{2}(\mathbf{E}\times\mathbf{H}^*)$ (Poynting flux), $w_e = \frac{1}{4}(\mathbf{E}\cdot\mathbf{D}^*)$, and $w_m = \frac{1}{4}(\mathbf{B}\cdot\mathbf{H}^*)$ (electric and magnetic energy density). Note that only the real parts of these quantities have physical significance. Your result is Poynting's theorem for periodic fields (This is essentially Jackson 6.134, but I want to see all steps in the derivation.)