PHYS 942 MIDTERM Exam

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PHYS 942
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253 DeMeritt

Name, please write clearly: ____________________

Note: Open book (Jackson). 250 points max, 100 are a perfect score! Please write clearly. **Show all your steps!**

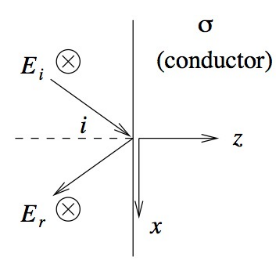
1. (*50 points*) Consider a rotating electric dipole $\mathbf{p}$ such that the dipole lies in the x-y plane and rotates about the z-axis with angular velocity ω. Calculate $dP/d\Omega$ as a function of distance r and the angle θ between the observer and the z-axis. Hint: You can construct a rotating dipole from two oscillating linear dipoles, which is most conveniently expressed as a complex dipole.

2. (*50 points*) Rectangular cavity:

   (a) Consider the TE mode in a rectangular cavity of dimensions $a \times b \times c$. Calculate the eigenfrequencies assuming perfectly conducting walls.

   (b) Calculate the fields for the eigenmode with the lowest frequency, assuming $a < b < c$.

3. (*50 points*) EM wave penetration: A plane polarized electromagnetic wave of frequency ω is incident with angle I on a flat surface of an excellent conductor ($\mu = \mu_0$, $\epsilon = \epsilon_b$, and $\sigma \gg \omega\epsilon_b$), which fills the region $z > 0$. Consider only linear polarization perpendicular to the plane of incidence as indicated in the figure.



   If the incident wave is given by $\mathbf{E} = \mathbf{E}_i e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$, show that the magnitude of the electric field inside the conductor is

$$E_c = E_i \gamma \cos I e^{-z/\delta} e^{i(kx \sin I + z/\delta - \omega t)}$$

   with $\delta = \sqrt{2/\omega\mu_0\sigma}$ and $\gamma = (1 - i)\sqrt{2\epsilon_0\omega/\sigma}$ (note that I capitalized the angle of incidence I to avoid confusion with the complex constant i).

4. (*50 points*) A dipole antenna of length d with its axis along the z-axis is excited in such a way that the sinosoidal current makes exactly one full wavelength oscillation over the length of the antenna, i.e., $k = 2\pi/d$.

   (a) Write down the current density $\mathbf{J}(z)$.

   (b) Calculate $\mathbf{A}(\mathbf{x})$ in the radiation zone with no other approximation than $d \ll \lambda \ll r$ (from Jackson, eq. 9.8) and express the result in terms of spherical coordinates (r, θ).

   (c) Calculate the magnetic field of the wave.

   (d) Calculate the radiated power $dP/d\Omega$.

   The identity $2 \sin\alpha \sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$ will come in handy.

5. (*50 points*) Group speed: In a medium where the elecrons experience no damping, the dispersion relation can be written as:

$$k = \frac{\omega}{c}\left[1 + \omega_p^2 \sum \frac{f_j}{(\omega_j^2 - \omega^2)}\right].$$

   Calculate the group velocity v_G and show that $V_G < c$ even when the phase velocity is larger than c.