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Tracking Magnetic Nulls in a Simulated Magnetosphere

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Abstract. We describe a technique for tracking magnetic nulls in computer simulations of magnetized plasmas. The technique is based on the Greene (1992) algorithm of computing the topological degree of a discretized vector field. We apply the Greene algorithm to a resistive magnetohydrodynamics (MHD) simulation of Earth's magnetosphere. We demonstrate that under generic northward interplanetary magnetic field (IMF) conditions, the large scale topology of the dayside magnetopause is consistent with a null-null separator topology, with the dayside X line extending across the subsolar magnetopause and terminating at two magnetic nulls in the polar cusps.

1. Introduction

The Dungey (1961, 1963) reconnecting magnetosphere model has proved to be a powerful framework within which to organize a large set of ground and space based observations (e.g., ionospheric convection patterns observed with ground based radar, *in situ* spacecraft observations of the precipitation of solar wind particles into the magnetosphere, etc. – see Kennel (1995) for a review). Nevertheless, the three-dimensional magnetic topology of the magnetosphere remains something of a puzzle. For example, when the interplanetary magnetic field (IMF) is due northward, Dungey (1963) visualized magnetospheric reconnection by projecting the magnetic field onto the noon-midnight meridional plane, as illustrated in Fig. 1. In such a projection, one naturally identifies the magnetic neutral points as potential sites of reconnection. In three dimensions, however, the magnetic field topology for exactly northward IMF differs qualitatively from that shown in Fig. 1. Figure 2 shows the magnetic field topology of a vacuum superposition (in which a uniform IMF is superimposed on a dipole field) for the case where Earth's dipole tilt is zero and the IMF is due northward. Unlike the situation in Fig. 1, in which there are two distinct separatrix surfaces which intersect to form two distinct null lines, in Fig. 2 there is only a single surface which separates solar wind field lines from closed magnetospheric field lines. The separatrix consists of an infinite number of field lines which join the two cusp nulls (shown as small spheres in Fig. 2). Further, the separatrix surface of Fig. 2 is structurally unstable, that is, the surface does not survive general (nonideal) perturbations of the magnetic field (see Fig. 2). In the generic case, in which the dipole and IMF axes are not aligned, the single separatrix surface of Figure 2 bifurcates into two distinct surfaces, and the reconnection "X line" is identified as the intersection of the two separatrix surfaces. In this paper, we apply a topological degree algorithm developed by Greene (1992) to compute the topo-

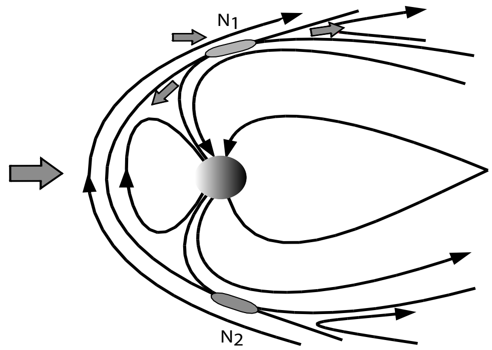

Figure 1. This figure illustrates reconnection in a two-dimensional magnetosphere under northward IMF conditions. The sun is to the left. The grey ovals are reconnection diffusion regions, localized around nulls N_1 and N_2. Grey arrows illustrate plasma flows, while black arrows illustrate the magnetic field.

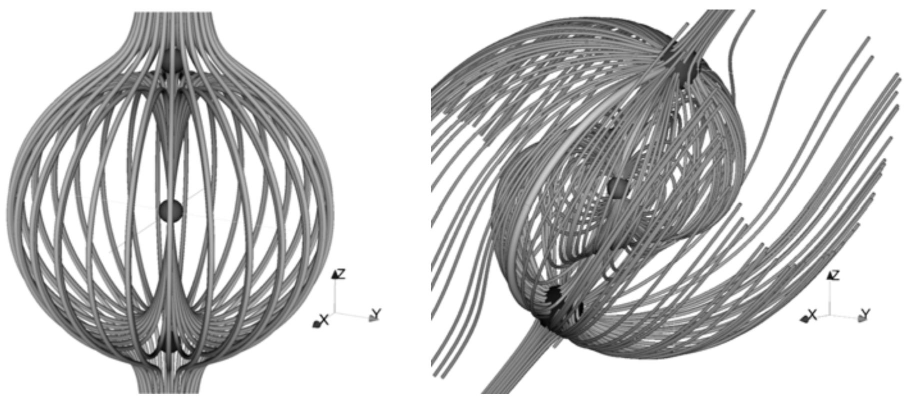

Figure 2. This figure shows a three-dimensional view of the vacuum superposition topology under pure northward IMF conditions (left) and when the interplanetary magnetic field is not antiparallel or parallel to the dipole axis (right). Nulls are illustrated by the two spheres in the polar cusps. The thick grey tube joining the two cusp nulls in the right figure is the magnetic separator, which defines the intersection of the Σ surfaces (illustrated here with field lines fanning out from each null)

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logy of Earth's dayside magnetopause as simulated by the OpenGGCM (Open Global Geospace Circulation Model) code (see Raeder 2003 for a description of the numerical methods used).

2. Tracking magnetic nulls in MHD simulations

Greene's algorithm is based on the concept of the *topological degree* of a map $f : R^n \rightarrow R^n$, relative to the domain $D \subseteq R^n$. The map f takes the vector $\mathbf{x} =< x_1, x_2, ..., x_n >$ to the vector $\mathbf{f} =< f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_n(\mathbf{x}) >$. Let $\mathbf{J}_f$ be the Jacobian of f. Then the topological degree (Kronecker 1869) of f, relative to domain D, is defined as

$$\deg(f, D) = \sum_{x \in f^{-1}(\mathbf{0})} \operatorname{sgn} \det(\mathbf{J}_f(x)), \tag{1}$$

where $\mathbf{0}$ is the n-dimensional zero vector. Thus, the topological degree is the difference between the number of solutions of $\mathbf{f} = \mathbf{0}$ for which $\det(\mathbf{J}_f(x)) > 0$ and the number for which $\det(\mathbf{J}_f(x)) < 0$. One can compute the topological degree by evaluating the Kronecker integral (see, for example, Polymilis et al. 2003):

$$\deg(f, D) = \frac{\Gamma(n/2)}{2\pi^{n/2}} \int_{\partial D} \frac{\sum_{i=0}^{n} A_i dx_1 ... dx_{i-1} dx_{i+1} ... dx_n}{(f_1^2 + f_2^2 + \cdots f_n^2)^{n/2}}, \tag{2}$$

where $\Gamma(x)$ is the gamma function and

$$A_i = (-1)^{n(i-1)} \det \mathbf{M} \tag{3}$$

Here $\mathbf{M}$ is the $n \times n$ matrix such that $M_{i1} = f_i$ and $M_{ij+1} = \frac{\partial f_i}{\partial x_j}$, with j ranging over $\{1, 2, ..., i-1, i+1, ..., n\}$.

For example, when the map in question is the magnetic field, $\mathbf{B}$, the Kronecker integral takes the following form:

$$\deg(\mathbf{B}, D) = \frac{1}{4\pi} \int_{\partial D_B} \frac{\mathbf{B} \cdot d\sigma}{B^3}, \tag{4}$$

where we have transformed the integral into magnetic field space, D_B is the image of D under the map $\mathbf{B}(\mathbf{x})$, $d\sigma$ is a differential surface element in magnetic field space, and B is the magnitude of the magnetic field (B is the distance from the origin in magnetic field space). By Gauss's law, if ∂D_B encloses the origin once (i.e., if ∂D contains a single magnetic null), then $\deg(B, D) = \pm 1$ (the sign is determined by the orientation of ∂D_B, which is, in turn, determined by the map $\mathbf{B}$ from D to D_B). Similarly, if D contains N nulls, then one can break up (4) into a sum of integrals, each one corresponding to a subvolume, D_s, enclosing a single null.

Greene (1992) discretizes (4) by sampling magnetic field vectors on D, triangulating the sampled points (see Fig. 3), transforming the resulting triangles into magnetic field space (thus, approximating ∂D_B by the polyhedron $\partial \tilde{D}_B$ in magnetic field space), and computing the following sum:

$$\deg(\mathbf{B}, D) \approx \sum_{i=0}^{N_T} A_i, \tag{5}$$

where N_{T} is the number of triangles, and

$$\begin{aligned} A_i &= 4 \arctan\{[\tan(\theta_1 + \theta_2 + \theta_3)/4 \\ &\times \tan(\theta_1 + \theta_2 - \theta_3)/4 \\ &\times \tan(\theta_2 + \theta_3 - \theta_1)/4 \\ &\times \tan(\theta_3 + \theta_1 - \theta_2)/4]^{1/2}\}. \end{aligned} \tag{6}$$

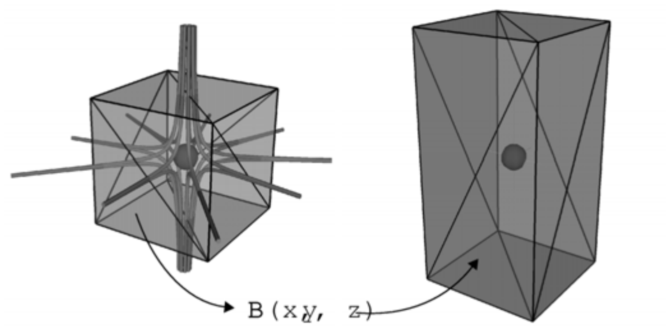

Figure 3. This figure illustrates the calculation of the topological degree of a discretized magnetic field, relative to an OpenGGCM finite difference cell. Each computational cell is decomposed into 12 triangles (left cube), each of which is mapped (using the values of the magnetic field at the eight vertices of the cell) to a corresponding triangle in magnetic field space (right cube). In this example, there is a single linear null (grey sphere) such that $B_x = x$, $B_y = y$, and $B_z = -2z$.

Here A_i is the area of the spherical triangle corresponding to the projection of the i-th triangle of $\partial\tilde{D}_B$ onto the unit sphere in magnetic field space: $\cos\theta_i = (\mathbf{B}_j \cdot \mathbf{B}_k)/|\mathbf{B}_j||\mathbf{B}_k|$, where the indices $\{i, j, k\}$ are cyclic permutations of $\{1, 2, 3\}$. The areas of the spherical triangles are oriented such that A_i has the same sign as the volume element $\mathbf{B}_i \cdot \mathbf{B}_j \times \mathbf{B}_k > 0$.

3. OpenGGCM simulations

Figure 4 shows the magnetic skeleton computed from the OpenGGCM simulation after 6840 seconds of simulated time. Steady solar wind boundary conditions, with a generic northward interplanetary magnetic field, were used to drive the simulated magnetosphere. The thin lines in the figure are magnetic field streamlines corresponding to 180 seed points randomly distributed within spheres of radii $1.5\,R_{\mathrm{E}}$ around the northern (marked “A”) and southern (marked “B”) cusp nulls, located at the point $N_1 = (-2.4\,R_{\mathrm{E}}, 6.3\,R_{\mathrm{E}}, 12.9\,R_{\mathrm{E}})$ and $N_2 = (-3.2\,R_{\mathrm{E}}, -6.5\,R_{\mathrm{E}}, -13.5\,R_{\mathrm{E}})$, respectively. Thus, the thin lines lie approximately on the Σ surfaces (here, we are following the nomenclature of Lau & Finn (1990)) associated with the two nulls. The thick grey line is the magnetic field streamline which passes through the point $(10.35\,R_{\mathrm{E}}, 0, 0)$, the approximate location of the magnetopause along the Sun-Earth line. Note that this line passes very close to the two nulls used to visualize the Σ surfaces. Also note that the two Σ surfaces come into contact at the approximate location of the thick grey field line. Thus, the thick line gives the approximate location of the magnetic separator defined by the intersection of the two separatrix surfaces associated with nulls N_1 and N_2.

It is clear from Fig. 4 that the topology of the simulated magnetopause is more complex than that of the simple vacuum superposition. While the vacuum superposition topology has two magnetic nulls, a single type A null and a single type B null, the topology shown in Fig. 4 has more than two magnetic nulls.

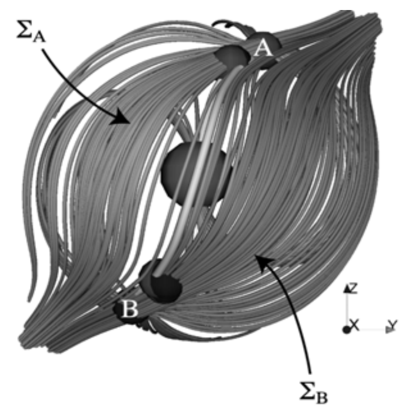

Figure 4. This figure shows the magnetic skeleton computed from the OpenGGCM simulation after 6840 seconds of simulated time. Nulls are illustrated as small spheres. The large central sphere is centered at Earth. The thin lines are magnetic field lines which lie approximately on the Σ surfaces: one set of lines fans out from the sphere marked "A" (a type A null), and the other set fans out from the "B" null. The thick grey line lies approximately at the intersection of the two Σ surfaces.

Indeed, there appear to be four distinct clusters of magnetic nulls, two in the northern polar cusp and two in the southern polar cusp. Nevertheless, while the number of nulls in each cluster varies in time (with nulls being created and destroyed in pairs), the locations of the clusters remain relatively steady. Further, if one computes the topological degree of each cluster, one finds that the large scale topology is consistent with a simple two-null separator topology. This is illustrated in Fig. 5, which shows the number of type A nulls (squares) and type B nulls (circles) within each cluster as a function of time. After a steady state has been reached (i.e., after about 1000 seconds of simulated time), the number of type A nulls in the northernmost cluster always exceeds the number of type B nulls by one; thus, this cluster has a topological degree of 1. In contrast, the southernmost cluster has a topological degree of -1. The two intermediate clusters always have equal numbers of type A and type B nulls, corresponding to a vanishing topological degree. Thus, the dayside magnetopause magnetic field topology is consistent, on the large scale, with a simple separator topology, despite the fact that the polar cusp topology is more complex (with multiple nulls in each polar cusp).

4. Conclusions

We have demonstrated the use of the Greene (1988) topological degree algorithm for tracking magnetic nulls in a resistive MHD simulation of Earth's magnetosphere. The simulation was carried out with the OpenGGCM code (see Raeder 2003 for a description of numerical methods implemented in the code). We considered a single simulation, corresponding to steady solar wind conditions with a generic northward IMF, and computed the topological degree of each computational cell of the simulation grid. A topological degree of 1 indicates that the cell

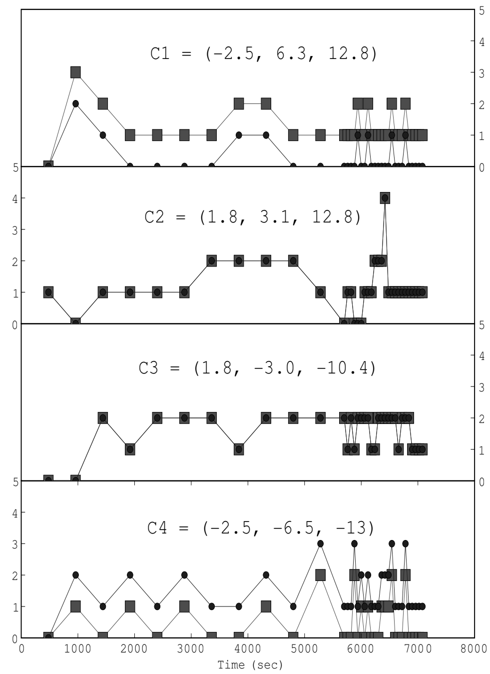

Figure 5. This figure shows the temporal variation of the dayside magnetopause magnetic field topology. The four panels show the number of magnetic nulls within spheres of radius $2\,R_{\mathrm{E}}$ centered at $C1$, $C2$, $C3$, and $C4$, respectively. Squares show the number of type A nulls, while circles show the number of type B nulls.

contains a type A null; a topological degree of -1 indicates that the cell contains a type B null. We found that the topology of the dayside magnetopause was consistent, on the large scale, with a simple null-null separator topology (see, for example, Lau & Finn 1990), but the field topology was more complex in the polar cusps, with multiple nulls forming and annihilating in pairs.

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