Global MHD Simulations: Nuts and Bolts

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Outline

- Some history of global modeling
- Geometry and grids
- Equations
- Initial and boundary conditions
- Ionosphere model
- MHD solver – algorithms
- Validation
- Computing issues
- To do list
A Brief History of Global MHD Simulations

- 1978: First 2d simulations by Leboeuf et al. (so we are close to the 20\textsuperscript{th} anniversary).
- Early 80’s: First 3d simulations (Brecht, Lyon, Wu, Ogino).
- Late 80’s: Model refinements (FACs, ionosphere, higher resolution, fewer symmetries).
- Early 90’s: Long geomagnetic tails, refined ionosphere models.
- Mid 90’s: ISTP is well underway, modeling has become part of the missions, first comparisons with \textit{in situ} space observations and ground based observations. Beginning of \textit{quantitative} modeling.
- Late 90’s: Global modeling has become an integrated part of many experimental studies. Models provide an extension to spatially limited observations and help us to understand the physics.
Simulation geometry and grid

- Simulation boundaries should be in supermagnetosonic flows, i.e., $\geq 18 \ R_E$ from Earth on the sunward side, $\geq 200 \ R_E$ in the tailward direction, and $\geq 50 \ R_E$ in the transverse directions.

- Numerical grids:

  ⇒ uniform cartesian: lowest programming overhead, lowest computing overhead, no memory overhead, easiest parallelization, near perfect load balancing, not adaptable

  ⇒ stretched cartesian: low programming overhead, low computing overhead, no memory overhead, easiest parallelization, near perfect load balancing, somewhat adaptable
grids (continued)

⇒ nested cartesian (can be self-adapting): medium to high programming overhead, small computing overhead, medium memory overhead, difficult to parallelize and load balance, internal discontinuities, very adaptable

⇒ non-cartesian with regular topology: medium programming overhead, small computing overhead, low memory overhead, parallelizes and load balances like regular cartesian grid, somewhat adaptable
grids (continued)

⇒ non-cartesian with irregular topology (can be self-adapting): high programming overhead, high computing overhead, high memory overhead, difficult to parallelize and load balance, smooth internal transitions, very adaptable, can use FEM technology

⇒ UCLA-GGCM grid:
Equations

- non-conservative (primitive variables)

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\frac{\partial \mathbf{v}}{\partial t} &= -(\mathbf{v} \cdot \nabla) \mathbf{v} - \left(\frac{1}{\rho}\right) \nabla p + \left(\frac{1}{\rho}\right) j \times \mathbf{B} \\
\frac{\partial p}{\partial t} &= -(\mathbf{v} \cdot \nabla) p - \gamma p \mathbf{\nabla} \cdot \mathbf{v} \\
\frac{\partial \mathbf{B}}{\partial t} &= -\mathbf{\nabla} \times \mathbf{E} \\
\nabla \cdot \mathbf{B} &= 0 \\
\mathbf{E} &= -\mathbf{v} \times \mathbf{B} + \eta \mathbf{j} \\
\mathbf{j} &= \mathbf{\nabla} \times \mathbf{B}
\end{align*}
\]

⇒ no strict numerical conservation of momentum and energy possible

⇒ numerical difficulties with convective derivatives

⇒ leads to numerical difficulties with strong shocks, errors in RH conditions and shock speed
Equations (continued)

- full conservative

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\frac{\partial \rho \mathbf{v}}{\partial t} &= -\nabla \cdot \{\rho \mathbf{v} \mathbf{v} + p \mathbb{I} - (\mathbf{B} \mathbf{B} - \frac{1}{2} \mathbf{B}^2 \mathbb{I})\} \\
\frac{\partial \mathbf{U}}{\partial t} &= -\nabla \cdot \{(U + p) \mathbf{v} + \mathbf{E} \times \mathbf{B}\} \\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E} \\
\nabla \cdot \mathbf{B} &= 0 \\
\mathbf{E} &= -\mathbf{v} \times \mathbf{B} + \eta \mathbf{j} \\
\mathbf{j} &= \nabla \times \mathbf{B} \\
p &= (\gamma - 1) \left\{U - \frac{1}{2} \rho \mathbf{v}^2 - \frac{1}{2} \mathbf{B}^2\right\}
\end{align*}
\]

⇒ allows strict numerical conservation of mass, momentum and energy

⇒ numerical difficulties in low $\beta$ regions (negative pressure possible because $p$ becomes difference of large numbers)
Equations (continued)

- Gas dynamic conservative

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho v) \\
\frac{\partial \rho v}{\partial t} &= -\nabla \cdot (\rho vv + p I) + j \times B \\
\frac{\partial e}{\partial t} &= -\nabla \cdot (\{e + p\} v) + j \cdot E \\
\frac{\partial B}{\partial t} &= -\nabla \times E \\
\nabla \cdot B &= 0 \\
E &= -v \times B + \eta j \\
j &= \nabla \times B \\
p &= (\gamma - 1)\{e - \frac{1}{2} \rho v^2\}
\end{align*}
\]

⇒ compromise

⇒ allows strict numerical conservation of mass, momentum and plasma energy, but no strict conservation of total energy

⇒ low \(\beta\) regions pose no difficulty

⇒ could be combined with full conservative scheme by integrating both energy equations and using a \`\beta\ switch'
Anomalous resistivity

• Current driven instabilities:

  ⇒ ion sound instability:

  \[ \eta \sim \left( \frac{c}{\omega_c} \right)^2 \omega_i \left( \frac{v_D}{c_s} \right) \left( \frac{T_e}{T_i} \right) \]

  ⇒ electron–cyclotron drift instability:

  \[ \eta \sim r_e^2 \Omega_e \left( \frac{v_D}{v_e} \right)^3 \]

  ⇒ lower hybrid drift instability:

  \[ \eta \sim r_e^2 \left( \frac{m_i}{m_e} \right) \left( \frac{v_D}{v_e} \right)^2 \omega_{lh} \]

  ⇒ MTSI/KCSI/IWI:

  \[ \frac{\nu_c}{\Omega_i} \approx 0.005 - 0.08 \quad \eta \approx 10^{-7} \quad \text{to} \quad 2 \times 10^{-5} s \]

  ⇒ simulation studies [Tanaka, Brackbill]: \( \eta \sim v_D^2 \)

  ⇒ observations [Cattell et al.]: \( R_m \approx 0.1 \ldots 10 \)

  ⇒ most known anomalous resistivity models predict \( \eta \sim j^p \) with \( p = 2 \) the most likely value

• Parameterization:

  \[ \eta = \alpha j^2 \quad \text{if} \quad j' \geq \delta, \quad 0 \quad \text{otherwise} \]

  \[ j' = \frac{|j| \Delta}{|B| + \epsilon} \]
Boundary conditions

- Sunward side:

  ⇒ Arbitray fixed or time dependent
  ⇒ Measured solar wind data
  ⇒ Problem with $B_x$: Three dimensional structure of the solar wind needs to be known because

  $$\nabla \cdot \mathbf{B} = 0 \iff \mathbf{n} \cdot (B_{upstream} - B_{downstream}) = 0$$

  ⇒ implies that $B_x = B_n$ cannot change if solar wind parameters are independent of $Y$ and $Z$.

  ⇒ solution: find $\mathbf{n}$, difficult with single solar wind monitor, boundary normal methods (minimum variance, ...) can be applied

- All other sides:

  ⇒ free flow conditions for plasma and transverse $\mathbf{B}$ components:

  $$\frac{\partial \Psi}{\partial \mathbf{n}} = 0$$

  ⇒ normal component of $\mathbf{B}$: follows from $\nabla \cdot \mathbf{B} = 0$

- Inner boundary (ionosphere): later
Initial conditions

- Magnetic field:
  Superposition of dipole with mirror dipole to create $B_x = 0$
surface sunward of Earth, then replace field on sunward side
with initial solar wind field.

- plasma:
  cold (5000 K), tenous (0.1 cm$^{-3}$), uniform
Ionosphere model

- Geometry and mapping:
  pick field aligned currents ($j_\parallel$), at the inner boundary (2 - 4 $R_E$) from the MHD grid and map along dipole field lines onto the ionosphere:

- covers latitudes from 58° to 90°.
- very high $v_A$ inside inner boundary, solving the MHD equations is not necessary.
- Use mapped FAC, precipitation parameters to solve for the ionospheric potential $\Phi$.
- Map potential back to inner boundary and use as boundary condition for flow:

$$\mathbf{v} = \frac{(-\nabla \Phi) \times \mathbf{B}}{|\mathbf{B}|^2}$$
Ionosphere model (continued)

- Limiting cases:
  \[ \Phi = 0 \implies E = 0 \implies v = 0 \] (equivalent to infinite ionospheric conductance): field lines are tied in the ionosphere, no convection in the ionosphere, and convection in the magnetosphere inhibited.

  \[ j_{\parallel} \to 0 \] (zero conductance): Field lines slip free through the ionosphere and Earth.

  \[ \text{in reality the ionosphere has a finite conductance and field lines are dragged through the ionospheric plasma, dissipating energy.} \]

- Potential equation, solved on each hemisphere separately:
  \[ \nabla \cdot \Sigma \cdot \nabla \Phi = -j_{\parallel} \sin I \]

- Boundary condition: \( \Phi(\text{equator}) = 0. \)

- Conductance tensor:
  \[ \Sigma = \begin{pmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta\lambda} \\ -\Sigma_{\theta\lambda} & \Sigma_{\lambda\lambda} \end{pmatrix} \]

  \[ \Sigma_{\theta\theta} = \frac{\Sigma_P}{\sin^2 I} , \quad \Sigma_{\theta\lambda} = \frac{\Sigma_H}{\sin I} , \quad \Sigma_{\lambda\lambda} = \Sigma_P \]
Ionosphere model (continued)

- Ionospheric conductances, 3 primary sources:
  - Solar EUV [Moen and Brekke, 1993]:
    \[
    \Sigma_H = F_{10.7}^{0.53}(0.81 \cos \chi + 0.54 \cos^{1/2} \chi) \\
    \Sigma_P = F_{10.7}^{0.49}(0.34 \cos \chi + 0.93 \cos^{1/2} \chi)
    \]
  - Parallel potential drops [Knight, 1973; Lyons et al., 1979]:
    \[
    \Delta \Phi = K \max(0, -j_\parallel) \\
    K = \frac{e^2 n_e}{\sqrt{2\pi m_e kT_e}}
    \]
  - Electron acceleration:
    \[
    F_E = \Delta \Phi j_\parallel, \quad E_0 = e \Delta \Phi
    \]
  - Pitch angle scattering:
    \[
    F_E = n_e (kT_e/2\pi m_e)^{1/2}, \quad E_0 = kT_e
    \]
  - Hall, Pedersen conductance from e - precipitation: [Hardy et al., 1987]:
    \[
    \Sigma_P = \left[40E_0/(16 + E_0^2)\right] F_E^{1/2} \\
    \Sigma_H = 0.45 E_0^{5/8} \Sigma_P
    \]
MHD numerics

- Time differencing

⇒ Model equation:
\[
\frac{\partial U}{\partial t} = -\nabla \cdot F(U)
\]

⇒ Explicit time differences, predictor - corrector scheme (second order accurate):
\[
U^{n+1} = U^n - \frac{1}{2} \Delta t \nabla \cdot F(U^n)
\]
\[
U^{n+1} = U^n - \Delta t \nabla \cdot F(U^{n+1})
\]

⇒ Explicit time differences, leap - frog scheme (second order accurate):
\[
U^{n+1} = U^{n-1} - 2\Delta t \nabla \cdot F(U^n, U^{n-1})
\]

⇒ Stability criterion (Courant-Fridrichs-Levy, CFL):
\[
\Delta t_{max} \leq \delta \min(\Delta x, \Delta y, \Delta z) \\frac{1}{|\nabla| + v_{MS}}
\]

⇒ CFL criterion can be very restrictive, \(\Delta t_{max}\) must be satisfied everywhere in the simulation domain

⇒ Implicit time differencing schemes:
\[
U^{n+1} = U^{n-1} - \Delta t \nabla \cdot F(U^{n+1}, U^n, U^{n-1}, \ldots)
\]

⇒ Implicit time differencing can be unconditionally stable, but generally requires the solution of large linear systems, too expensive and impractical
MHD numerics (continued)

• Spatial discretization:
  ⇒ Finite differences (FD).
  ⇒ Finite volume (FD), reduces to FD on cartesian grids.
  ⇒ Finite element (FEM), mostly used for non-cartesian irregular grids.

• Conservative FD:
  ⇒ Model equation:
    \[
    \frac{\partial U}{\partial t} = -\nabla \cdot F(U)
    \]
  ⇒ discretize as:
    \[
    \frac{\partial U}{\partial t} = - \left( f_{i+\frac{1}{2}\,j}(U) - f_{i-\frac{1}{2}\,j}(U) \right)/\Delta x \\
    - \left( f_{i\,j+\frac{1}{2}}(U) - f_{i\,j-\frac{1}{2}}(U) \right)\Delta y
    \]
  ⇒ numerical fluxes:
    \[
    f_{i+\frac{1}{2}\,j} = G(\ldots, U_{i-1,j}, U_{i,j}, U_{i+1,j}, \ldots) \\
    f_{i\,j+\frac{1}{2}} = G(\ldots, U_{i,j-1}, U_{i,j}, U_{i,j+1}, \ldots)
    \]
  ⇒ numerical fluxes must be consistent with the physical flux \( F(U) \):
    \[
    G(U, \ldots, U, U, \ldots, U) = F(U)
    \]
MHD numerics (continued)

- Schematic of conservative FD:

  \[ U_{i-1,j+1} \quad U_{i,j+1} \quad U_{i+1,j+1} \]
  \[ \circ \quad \circ \quad \circ \]
  \[ f_{i,j+\frac{1}{2}} \]
  \[ U_{i-1,j} \quad f_{i-\frac{1}{2},j} \quad U_{i,j} \quad f_{i+\frac{1}{2},j} \quad U_{i+1,j} \]
  \[ \circ \quad \bullet \quad \circ \quad \bullet \quad \circ \]
  \[ f_{i,j-\frac{1}{2}} \]
  \[ U_{i-1,j-1} \quad U_{i,j-1} \quad U_{i+1,j-1} \]
  \[ \circ \quad \circ \quad \circ \]

- This differencing is equivalent to (and therefore conservative):

  \[
  \frac{\partial}{\partial t} \int \int \int_V U \, dV = \int \int_S F \, ds
  \]
MHD numerics (continued)

• Examples of numerical fluxes:

  ⇒ second order central:

\[ f_{i+\frac{1}{2}} = \frac{1}{2}(F(U_i) + F(U_{i+1})) \]

  ⇒ fourth order central:

\[ f_{i+\frac{1}{2}} = \frac{7}{12}(F(U_i) + F(U_{i+1})) - \frac{1}{12}(F(U_{i-1}) + F(U_{i+2})) \]

  ⇒ Lax scheme:

\[ f_{i+\frac{1}{2}} = \frac{1}{2}(F(U_i) + F(U_{i+1})) - \frac{1}{2}(U_{i+1} - U_i) \]

  ⇒ Two step Lax Wendroff scheme: Use Lax scheme for predictor, and second order central for corrector.

  ⇒ Rusanov scheme:

\[ f_{i+\frac{1}{2}} = \frac{1}{2}(F(U_i) + F(U_{i+1})) - \frac{1}{4}(|v_{i+1}| + |v_i| + c_i + c_{i+1})(U_{i+1} - U_i) \]

  ⇒ Godunov schemes: solve a Riemann problem (i.e. the decay of a step function into waves) at the cell interface and compute the fluxes directly from the wave propagation. Accurate for gas-dynamics, but difficulties in MHD: degenerate eigenvector because of \( \nabla \cdot B = 0 \).
MHD numerics (continued)

- Error terms:

\[
\Delta x \frac{\partial U}{\partial t} = -(f_i + \frac{1}{2} - f_i - \frac{1}{2}) + a_1 (\Delta x)^2 \frac{\partial^2 F(U)}{\partial x^2} + b_1 (\Delta x)^3 \frac{\partial^3 F(U)}{\partial x^3} \\
+ a_2 (\Delta x)^4 \frac{\partial^4 F(U)}{\partial x^4} + b_2 (\Delta x)^5 \frac{\partial^5 F(U)}{\partial x^5} + \ldots
\]

⇒ error terms with even derivatives cause diffusion
⇒ error terms with odd derivatives cause dispersion
⇒ central difference schemes have no diffusion, but dispersion, big problem at shocks and discontinuities
⇒ first order schemes are less dispersive, but very diffusive
⇒ see examples: wiggles at discontinuities

- Monotonicity: A scheme is called monotone if it lets no new extrema develop in the solution (that is exactly what we want).
Harten's (Di)Lemma:  
* A monotone scheme is at most first order accurate! 
Thus, a globally monotone scheme will always be very diffusive.

Solution to Harten's (Di)Lemma:  
Hybridize the numerical fluxes: Use first order numerical flux ($f^l$) where a new extremum might develop (like at a shock), and use high order fluxes ($f^h$) where the solution is smooth.

\[ f_{i+\frac{1}{2}} = \theta_{i+\frac{1}{2}} f^h_{i+\frac{1}{2}} + (1 - \theta_{i+\frac{1}{2}}) f^l_{i+\frac{1}{2}} \]

⇒ Obviously, $0 \leq \theta \leq 1$. $\theta$ can be a function of anything, but generally depends on gradients of the solution.

The switch function $\theta$ is called *Flux Limiter*. There is no optimal flux limiter. A few choices:

⇒ Hartens “edge condition” flux limiter

⇒ vanLeer's flux limiter

⇒ Flux Corrected Transport (FCT)

⇒ Total Variance Diminishing (TVD) schemes (the monotonicity constraint is somewhat relaxed)

⇒ ...
MHD numerics (continued)

• Keeping divergence of $B$ zero

$\Rightarrow \nabla \cdot B = 0$ is an initial condition, $\nabla \cdot B$ is conserved by Faraday's law:

$$\nabla \cdot \frac{\partial B}{\partial t} = \frac{\partial (\nabla \cdot B)}{\partial t} = -\nabla \cdot \nabla \times E = 0$$

$\Rightarrow \nabla \cdot B$ cleaning, projection method: solve

$$\nabla^2 \Psi = - (\nabla \cdot B)$$

and correct the field:

$$B' = B + \nabla \Psi$$

requires the numerical solution of a Poisson equation (expensive). Can only be as good as the solution of $\Psi$.

$\Rightarrow \nabla \cdot B$ convection: Effectively modify equations so that $\nabla \cdot B$ convects through the system (Gombosi):

$$\frac{d(\nabla \cdot B)}{dt} = 0$$

$\nabla \cdot B$ must convect out of the system (inner magnetosphere?).

$\Rightarrow$ Use numerical $\nabla \cdot$ and $\nabla \times$ operators with $\nabla \cdot \nabla \times \Psi = 0$

$\Rightarrow$ Use a magnetic flux conservative scheme that keeps $\nabla \cdot B = 0$ to roundoff error.
MHD numerics (continued)

⇒ magnetic flux conservative scheme (continued)
place the magnetic field components on the center of cell faces:

\[(B_x)_{i+\frac{1}{2}j,k}, \ (B_y)_{i,j+\frac{1}{2}k}, \ (B_z)_{i,j,k+\frac{1}{2}}\]

⇒ and the electric field (a numerical flux) on the centers of the cell edges:

\[(E_x)_{i,j+\frac{1}{2}k+\frac{1}{2}}, \ (E_y)_{i+\frac{1}{2}j,k+\frac{1}{2}}, \ (E_z)_{i+\frac{1}{2}j,k+\frac{1}{2}},\]

\[\text{[Evans and Hawley, 1987].}\]
MHD numerics (continued)

⇒ magnetic flux conservative scheme (continued)
then:

\[
\frac{\partial}{\partial t}(B_x)_{i+\frac{1}{2}, j, k} = \\
\frac{\{(E_y)_{i+\frac{1}{2}, j, k+\frac{1}{2}} - (E_y)_{i+\frac{1}{2}, j, k-\frac{1}{2}}\}}{\Delta z} \\
- \frac{\{(E_z)_{i+\frac{1}{2}, j+\frac{1}{2}, k} - (E_z)_{i-\frac{1}{2}, j+\frac{1}{2}, k}\}}{\Delta y}
\]

⇒ analogous for \(B_y\) and \(B_z\)

⇒ By advancing the field components in this way on all 6 cell faces and summing up it follows:

\[
\frac{\partial}{\partial t}\iint_{\text{cell}} \Phi df = \\
\Delta y\Delta z\left(\frac{\partial B_x}{\partial t}\right)_{i-\frac{1}{2}} + \Delta y\Delta z\left(\frac{\partial B_x}{\partial t}\right)_{i+\frac{1}{2}} + \Delta x\Delta z\left(\frac{\partial B_y}{\partial t}\right)_{j+\frac{1}{2}} + \ldots
\]

\[
= \{(E_y)_{i+\frac{1}{2}, j+\frac{1}{2}, k} - (E_y)_{i+\frac{1}{2}, j, k+\frac{1}{2}}\} + \\
\{(E_y)_{i+\frac{1}{2}, j, k-\frac{1}{2}} - (E_y)_{i+\frac{1}{2}, j, k}\} + \ldots \Delta x\Delta y\Delta z
\]

thus \(\Phi = \text{const.}\)

⇒ The field can be initialized divergence free by using a vector potential \(A\) in place of \(E\).
MHD numerics (continued)

- How to handle stretched grids:

  ⇒ grid coordinates given by analytic function:
  \[ x(i, j, k), \; y(i, j, k), \; z(i, j, k), \] then:

  \[
  \frac{\partial}{\partial x} F(x, y, z) = \frac{\partial F}{\partial i} \frac{\partial i}{\partial x} + \frac{\partial F}{\partial j} \frac{\partial j}{\partial x} + \frac{\partial F}{\partial k} \frac{\partial k}{\partial x}
  \]

  \[
  = \frac{\partial F}{\partial i} (\frac{\partial i}{\partial x})^{-1} + \frac{\partial F}{\partial j} (\frac{\partial j}{\partial x})^{-1} + \frac{\partial F}{\partial k} (\frac{\partial k}{\partial x})^{-1}
  \]

  ⇒ analogous for \( y \) and \( z \) derivative.

  ⇒ particularly simple for stretched grid:

  \[
  \frac{\partial}{\partial x} F(x, y, z) = \frac{\partial F}{\partial i} (\frac{\partial i}{\partial x})^{-1}
  \]

  \[
  \frac{\partial}{\partial y} F(x, y, z) = \frac{\partial F}{\partial j} (\frac{\partial j}{\partial y})^{-1}
  \]

  \[
  \frac{\partial}{\partial z} F(x, y, z) = \frac{\partial F}{\partial k} (\frac{\partial k}{\partial z})^{-1}
  \]
Validation

• Compare with non-trivial solutions of the MHD equations (few available):

⇒ shock tube problems (check for RH conditions, expansion shocks, flux limiters). Table of shock tube parameters used by ISTP:

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• B convection test to evaluate \( \nabla \cdot B = 0 \): initialize a non-trivial B field from \( \nabla \times A \) such that \( B = 0 \) in most of the domain (magnetic bubble) and convect with \( \mathbf{V} = (V_x, 0, 0) \). \( \nabla \cdot B = 0 \) violation will become evident by \( B_x \) sticking to the grid:

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

⇒

\[
\frac{d \mathbf{B}}{dt} = \mathbf{V} (\nabla \cdot B)
\]

• Convergence test: run same problem twice: with highest resolution possible and with factor two lower resolution. The results should be qualitatively the same and quantitatively similar.

• Reality test: drive model with observed solar wind and IMF and compare results with \textit{in situ} observations.
Computer issues

- Parallel implementation on MIMD (Multiple Instruction - Multiple Data) computers:

  ⇒ domain decomposition, guard cells, message passing:

  ⇒ extra nodes for: ionosphere(1), I/O(1), restart file manager(1), and for things to come
Computer issues (continued)

⇒ runs on IBM/SP2, CRAY/T3E, workstation (Beowulf) cluster, ...

⇒ language: f77, macro preprocessor, spag, ftncheck.

⇒ f90/HPF not mature enough and/or generally available, would require rewriting the code.

⇒ using MPI message passing library, either native or MPICH.

⇒ excellent scalability:

⇒ computing chore: \(~\) 2000 Flop/gridpoint/timestep.

⇒ current machines: \(~\) 100 MFlop/s.

⇒ realtime: 60 nodes, \(10^6\) cells.

⇒ output: several GB per run.
Things to do, given unlimited time and resources

- Adding more physics: inner magnetosphere, ring current

⇒ bounce averaged drift equations:

\[
\frac{d\lambda_k}{dt} = (\frac{\partial}{\partial t} + \mathbf{v}_k \cdot \nabla)\lambda_k = 0 \\
\frac{d\eta_k}{dt} = (\frac{\partial}{\partial t} + \mathbf{v}_k \cdot \nabla)\eta_k = 0
\]

\[
\mathbf{v}_k = B^{-2}q_k^{-1}(q_k\mathbf{E} \times \mathbf{B} + \lambda_k \mathbf{B} \times \nabla V^{-2/3}) \\
\lambda_k = W_k V^{2/3} \\
\eta_k = n_k V \\
V = \int ds / B
\]

[Harel et al., 1981; Wolf et al., 1983; 1991; 1993]

⇒ magnetic field: dipole, ..... 

⇒ plasma represented by macroparticles of constant \( \lambda_k \) and \( \eta_k \)

⇒ 32 log spaced energy levels, 0.1 to 300 keV

⇒ currently one species: protons

⇒ plasma loss due to pitch angle scattering: 10% of strong pitch angle scattering limit for protons

⇒ plasma loss due to charge exchange with exospheric hydrogen:[Smith and Bewtra, 1978; Anderson et al., 1987]
Things to do ..... (continued)

- Inner magnetosphere (continued)
  
  ⇒ plasma loss due to Coulomb drag with plasmaspheric electrons: [Fok et al., 1991]

  ⇒ 10° by 0.1 $R_E$ grid in the magnetic equator, $1.2 \leq L \leq 7$

  ⇒ 0.1 to 0.5 million macroparticles per species, 1 processor

  ⇒ long timesteps ($\approx 1$ minute)
Things to do ..... (continued)

- Coupling with thermosphere - ionosphere circulation models (CTIM, TIECGM, .....)

  ⇒ need: ionospheric parameters ($j_\parallel$, precipitation)

  ⇒ provide: better ionospheric conductances, potential (neutral wind coupling, flywheel effect), ionospheric outflow.

- Resolution, resolution, resolution

  ⇒ basic scaling law: $T \sim N^4 = h^{-4}$, but computer power grows exponentially with time: