Global MHD Simulations: Nuts and Bolts

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Outline

- Some history of global modeling
- Geometry and grids
- Equations
- Initial and boundary conditions
- Ionosphere model
- MHD solver – algorithms
- Validation
- Computing issues
- To do list

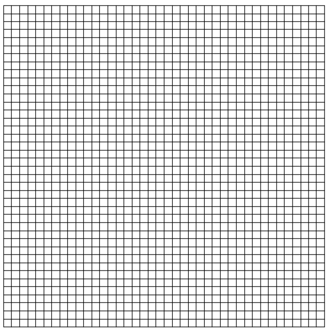
A Brief History of Global MHD Simulations

- 1978: First 2d simulations by Leboeuf et al. (so we are close to the 20^{th} anniversary).
- Early 80's: First 3d simulations (Brecht, Lyon, Wu, Ogino).
- Late 80's: Model refinements (FACs, ionosphere, higher resolution, fewer symmetries).
- Early 90's: Long geomagnetic tails, refined ionosphere models.
- Mid 90's: ISTP is well underway, modeling has become part of the missions, first comparisons with *in situ* space observations and ground based observations. Beginning of *quantitative* modeling.
- Late 90's: Global modeling has become an integrated part of many experimental studies. Models provide an extension to spatially limited observations and help us to understand the physics.

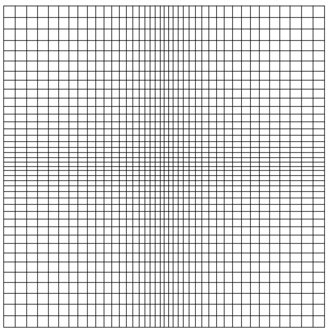
Simulation geometry and grid

- Simulation boundaries should be in supermagnetosonic flows, i.e., $\geq$ 18 R_E from Earth on the sunward side, $\geq$ 200 R_E in the tailward direction, and $\geq$ 50 R_E in the transverse directions.
- Numerical grids:

⇒ uniform cartesian: lowest programming overhead, lowest computing overhead, no memory overhead, easiest parallelization, near perfect load balancing, not adaptable

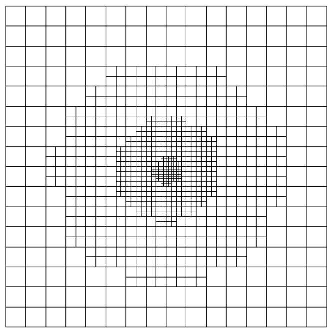


⇒ stretched cartesian: low programming overhead, low computing overhead, no memory overhead, easiest parallelization, near perfect load balancing, somewhat adaptable

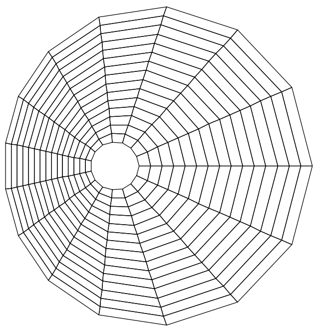


grids (continued)

⇒ nested cartesian (can be self-adapting): medium to high programming overhead, small computing overhead, medium memory overhead, difficult to parallelize and load balance, internal discontinuities, very adaptable

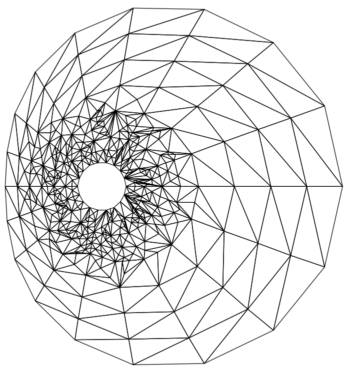


⇒ non-cartesian with regular topology: medium programming overhead, small computing overhead, low memory overhead, parallelizes and load balances like regular cartesian grid, somewhat adaptable

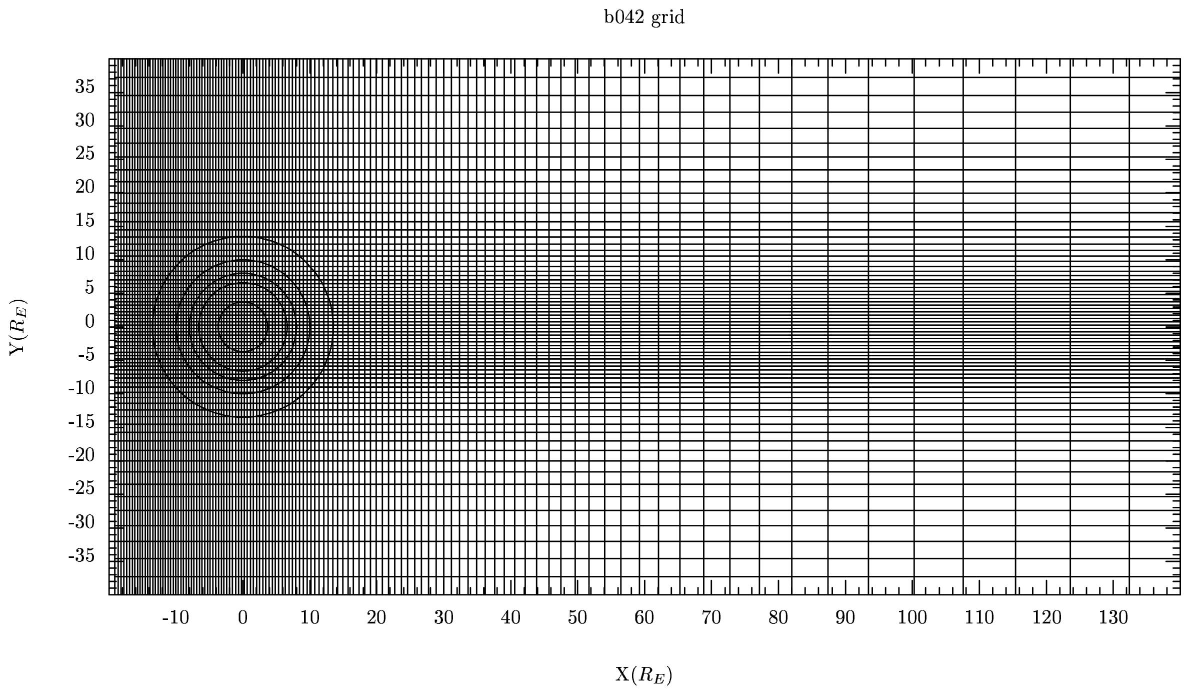


grids (continued)

⇒ non-cartesian with irregular topology (can be self-adapting): high programming overhead, high computing overhead, high memory overhead, difficult to parallelize and load balance, smooth internal transitions, very adaptable, can use FEM technology



⇒ UCLA-GGCM grid:



Equations

- non-conservative (primitive variables)

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\frac{\partial \mathbf{v}}{\partial t} &= -(\mathbf{v} \cdot \nabla)\mathbf{v} - (\tfrac{1}{\rho})\nabla p + (\tfrac{1}{\rho})\mathbf{j}\times\mathbf{B} \\
\frac{\partial p}{\partial t} &= -(\mathbf{v} \cdot \nabla)p - \gamma p \nabla \cdot \mathbf{v} \\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla\times\mathbf{E} \\
\nabla \cdot \mathbf{B} &= 0 \\
\mathbf{E} &= -\mathbf{v}\times\mathbf{B} + \eta\mathbf{j} \\
\mathbf{j} &= \nabla\times\mathbf{B}
\end{aligned}
$$

⇒ no strict numerical conservation of momentum and energy possible

⇒ numerical difficulties with convective derivatives

⇒ leads to numerical difficulties with strong shocks, errors in RH conditions and shock speed

Equations (continued)

- full conservative

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\frac{\partial \rho \mathbf{v}}{\partial t} &= -\nabla \cdot \{\rho \mathbf{v}\mathbf{v} + p\underline{\underline{\mathbf{I}}} - (\mathbf{B}\mathbf{B} - \tfrac{1}{2}B^2\underline{\underline{\mathbf{I}}})\} \\
\frac{\partial U}{\partial t} &= -\nabla \cdot \{(U + p)\mathbf{v} + \mathbf{E}\times\mathbf{B}\} \\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla\times\mathbf{E} \\
\nabla \cdot \mathbf{B} &= 0 \\
\mathbf{E} &= -\mathbf{v}\times\mathbf{B} + \eta\mathbf{j} \\
\mathbf{j} &= \nabla\times\mathbf{B} \\
p &= (\gamma - 1)\left\{U - \tfrac{1}{2}\rho v^2 - \tfrac{1}{2}B^2\right\}
\end{aligned}
$$

$\Rightarrow$ allows strict numerical conservation of mass, momentum and energy

$\Rightarrow$ numerical difficulties in low β regions (negative pressure possible because p becomes difference of large numbers)

Equations (continued)

- Gas dynamic conservative

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\frac{\partial \rho \mathbf{v}}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}\mathbf{v} + p\underline{\underline{\mathbf{I}}}) + \mathbf{j}\times\mathbf{B} \\
\frac{\partial e}{\partial t} &= -\nabla \cdot (\{e + p\}\mathbf{v}) + \mathbf{j}\cdot\mathbf{E} \\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla\times\mathbf{E} \\
\nabla \cdot \mathbf{B} &= 0 \\
\mathbf{E} &= -\mathbf{v}\times\mathbf{B} + \eta\mathbf{j} \\
\mathbf{j} &= \nabla\times\mathbf{B} \\
p &= (\gamma - 1)\{e - \tfrac{1}{2}\rho v^2\}
\end{aligned}
$$

$\Rightarrow$ compromise

$\Rightarrow$ allows strict numerical conservation of mass, momentum and plasma energy, but no strict conservation of total energy

$\Rightarrow$ low β regions pose no difficulty

$\Rightarrow$ could be combined with full conservative scheme by integrating both energy equations and using a `β switch'

Anomalous resistivity

- Current driven instabilities:

  ⇒ ion sound instability:

$$\eta \sim (c/\omega_e)^2 \omega_i (v_D/c_s)(T_e/T_i)$$

  ⇒ electron–cyclotron drift instability:

$$\eta \sim r_e^2 \Omega_e (v_D/v_e)^3$$

  ⇒ lower hybrid drift instability:

$$\eta \sim r_e^2 (m_i/m_e)(v_D/v_e)^2 \omega_{lh}$$

  ⇒ MTSI/KCSI/IWI:

$$\nu_c/\Omega_i \approx 0.005 - 0.08 \qquad \eta \approx 10^{-7} \quad to \quad 2{\times}10^{-5} s$$

  ⇒ simulation studies [Tanaka, Brackbill]: $\eta \sim v_D^2$

  ⇒ observations [Cattell et al.]: $R_m \approx 0.1 \ldots 10$

  ⇒ most known anomalous resistivity models predict $\eta \sim j^p$ with $p = 2$ the most likely value

- Parameterization:

$$\eta = \alpha j'^2 \quad \text{if} \quad j' \geq \delta, \quad 0 \ \text{otherwise} \tag{1}$$

$$j' = \frac{|j|\Delta}{|B| + \epsilon} \tag{2}$$

Boundary conditions

- Sunward side:

  $\Rightarrow$ Arbitray fixed or time dependent

  $\Rightarrow$ Measured solar wind data

  $\Rightarrow$ Problem with B_x: Three dimensional structure of the solar wind needs to be known because

$$\nabla \cdot \mathbf{B} = 0 \Longleftrightarrow \mathbf{n} \cdot (B_{upstream} - B_{downstream}) = 0$$

  $\Rightarrow$ implies that B_x = B_n cannot change if solar wind parameters are independent of Y and Z.

  $\Rightarrow$ solution: find $\mathbf{n}$, difficult with single solar wind monitor, boundary normal methods (minimum variance, ...) can be applied

- All other sides:

  $\Rightarrow$ free flow conditions for plasma and transverse $\mathbf{B}$ components:

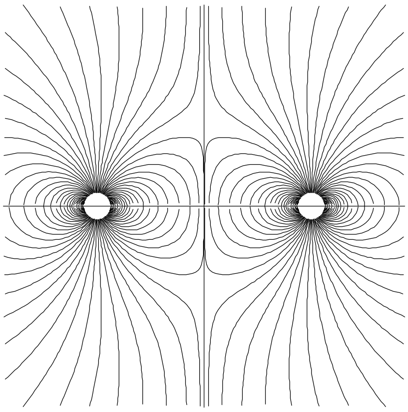
$$\frac{\partial \Psi}{\partial \mathbf{n}} = 0$$

  $\Rightarrow$ normal component of $\mathbf{B}$: follows from $\nabla \cdot \mathbf{B} = 0$

- Inner boundary (ionosphere): later

Initial conditions

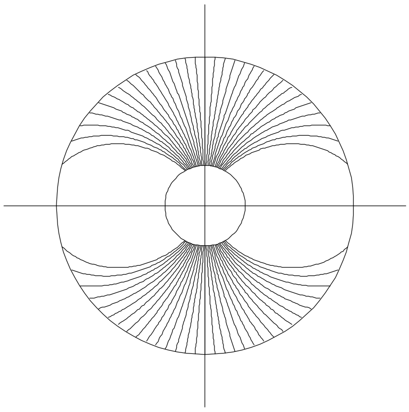
- Magnetic field:
  Superposition of dipole with mirror dipole to create $B_x = 0$ surface sunward of Earth, then replace field on sunward side with initial solar wind field.



- plasma:
  cold (5000 ^{0}K), tenous (0.1cm^{-3}), uniform

Ionosphere model

- Geometry and mapping:
  pick field aligned currents ($j_{\|}$), at the inner boundary (2 - 4 R_E) from the MHD grid and map along dipole field lines onto the ionosphere:



- covers latitudes from 58^0 to 90^0.
- very high v_A inside inner boundary, solving the MHD equations is not necessary.
- Use mapped FAC, precipitation parameters to solve for the ionospheric potential Φ.
- Map potential back to inner boundary and use as boundary condition for flow:

$$\mathbf{v} = \frac{(-\nabla\Phi)\times\mathbf{B}}{|B|^2}$$

Ionosphere model (continued)

- Limiting cases:

  $\Rightarrow$ $\Phi = 0 \quad \Longrightarrow \quad \mathbf{E} = 0 \quad \Longrightarrow \quad \mathbf{v} = 0$ (equivalent to infinite ionospheric conductance): field lines are tied in the ionosphere, no convection in the ionosphere, and convection in the magnetosphere inhibited.

  $\Rightarrow$ $j_\parallel \to 0$ (zero conductance): Field lines slip free through the ionosphere and Earth.

  $\Rightarrow$ in reality the ionosphere has a finite conductance and field lines are dragged through the ionospheric plasma, dissipating energy.

- Potential equation, solved on each hemisphere separately:

$$\nabla \cdot \underline{\underline{\mathbf{\Sigma}}} \cdot \nabla \Phi = -j_\parallel \sin I$$

- Boundary condition: Φ(equator)=0.

- Conductance tensor:

$$\underline{\underline{\mathbf{\Sigma}}} = \begin{pmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta\lambda} \\ -\Sigma_{\theta\lambda} & \Sigma_{\lambda\lambda} \end{pmatrix}$$

$$\Sigma_{\theta\theta} = \frac{\Sigma_P}{\sin^2 I} \quad , \quad \Sigma_{\theta\lambda} = \frac{\Sigma_H}{\sin I} \quad , \quad \Sigma_{\lambda\lambda} = \Sigma_P$$

Ionosphere model (continued)

- Ionospheric conductances, 3 primary sources:

  ⇒ Solar EUV [*Moen and Brekke, 1993*]:

$$\begin{aligned}\Sigma_H &= F_{10.7}^{0.53}(0.81\cos\chi + 0.54\cos^{1/2}\chi)\\ \Sigma_P &= F_{10.7}^{0.49}(0.34\cos\chi + 0.93\cos^{1/2}\chi)\end{aligned}$$

  ⇒ Parallel potential drops [*Knight, 1973; Lyons et al., 1979*]:

$$\Delta\Phi = K\max(0, -j_\parallel)$$

$$K = \frac{e^2 n_e}{\sqrt{2\pi m_e k T_e}}$$

  ⇒ Electron acceleration:

$$F_E = \Delta\Phi_\parallel j_\parallel \qquad , \qquad E_0 = e\Delta\Phi_\parallel$$

  ⇒ Pitch angle scattering:

$$F_E = n_e(kT_e/2\pi m_e)^{\frac{1}{2}} \qquad , \qquad E_0 = kT_e$$

  ⇒ Hall, Pedersen conductance from e - precipitation: [*Hardy et al., 1987*]:

$$\Sigma_P = [40E_0/(16+E_0^2)]F_E^{1/2}$$

$$\Sigma_H = 0.45E_0^{5/8}\Sigma_P$$

MHD numerics

- Time differencing

⇒ Model equation:

$$\frac{\partial U}{\partial t} = -\nabla \cdot \mathbf{F}(U)$$

⇒ Explicit time differences, predictor - corrector scheme (second order accurate):

$$U^{n+\frac{1}{2}} = U^n - \frac{1}{2}\Delta t \nabla \cdot \mathbf{F}(U^n)$$

$$U^{n+1} = U^n - \Delta t \nabla \cdot \mathbf{F}(U^{n+\frac{1}{2}})$$

⇒ Explicit time differences, leap - frog scheme (second order accurate):

$$U^{n+1} = U^{n-1} - 2\Delta t \nabla \cdot \mathbf{F}(U^n, U^{n-1})$$

⇒ Stability criterion (Courant-Fridrichs-Levy, CFL):

$$\Delta t_{max} \leq \delta \frac{\min(\Delta x, \Delta y, \Delta z)}{|\mathbf{v}| + v_{MS}}$$

⇒ CFL criterion can be very restrictive, Δt_{max} must be satisfied everywhere in the simulation domain

⇒ Implicit time differencing schemes:

$$U^{n+1} = U^{n-1} - \Delta t \nabla \cdot \mathbf{F}(U^{n+1}, U^n, U^{n-1}, \ldots)$$

⇒ Implicit time differrencing can be unconditionally stable, but generally requires the solution of large linear systems, too expensive and impractible

MHD numerics (continued)

- Spatial discretization:
  - ⇒ Finite differences (FD).
  - ⇒ Finite volume (FD), reduces to FD on cartesian grids.
  - ⇒ Finite element (FEM), mostly used for non - cartesian irregular grids.
- Conservative FD:
  - ⇒ Model equation:

$$\frac{\partial U}{\partial t} = -\nabla \cdot \mathbf{F}(U)$$

  - ⇒ discretize as:

$$\begin{aligned}\frac{\partial U}{\partial t} = - & \quad (f_{i+\frac{1}{2},j}(U) - f_{i-\frac{1}{2},j}(U))/\Delta x \\ - & \quad (f_{i,j+\frac{1}{2}}(U) - f_{i,j-\frac{1}{2}}(U))\Delta y\end{aligned}$$

  - ⇒ numerical fluxes:

$$f_{i+\frac{1}{2},j} = G(\ldots, U_{i-1,j}, U_{i,j}, U_{i+1,j}, \ldots)$$

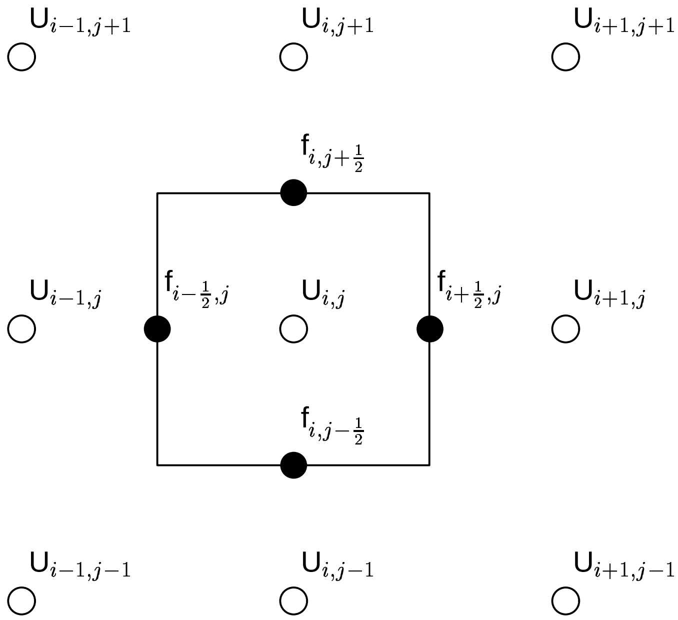
$$f_{i,j+\frac{1}{2}} = G(\ldots, U_{i,j-1}, U_{i,j}, U_{i,j+1}, \ldots)$$

  - ⇒ numerical fluxes must be consistent with the physical flux $F(U)$:

$$G(U, \ldots, U, U, \ldots, U) = F(U)$$

MHD numerics (continued)

- Schematic of conservative FD:



- This differencing is equivalent to (and therefore conservative):

$$\frac{\partial}{\partial t}\int\int\int_V U\,dV = \int\int_S \mathbf{F}\,d\mathbf{s}$$

MHD numerics (continued)

- Examples of numerical fluxes:

  $\Rightarrow$ second order central:

$$f_{i+\frac{1}{2}} = \frac{1}{2}(F(U_i) + F(U_{i+1}))$$

  $\Rightarrow$ fourth order central:

$$\begin{aligned} f_{i+\frac{1}{2}} \quad &= \tfrac{7}{12}(F(U_i) + F(U_{i+1})) \\ &-\tfrac{1}{12}(F(U_{i-1}) + F(U_{i+2})) \end{aligned}$$

  $\Rightarrow$ Lax scheme:

$$f_{i+\frac{1}{2}} = \frac{1}{2}(F(U_i) + F(U_{i+1})) - \frac{1}{2}(U_{i+1} - U_i)$$

  $\Rightarrow$ Two step Lax Wendroff scheme: Use Lax scheme for predictor, and second order central for corrector.

  $\Rightarrow$ Rusanov scheme:

$$\begin{aligned} f_{i+\frac{1}{2}} = \quad & \tfrac{1}{2}(F(U_i) + F(U_{i+1})) \\ & -\tfrac{1}{4}(|v_i| + |v_{i+1}| + c_i + c_{i+1})(U_{i+1} - U_i) \end{aligned}$$

  $\Rightarrow$ Godunov schemes: solve a Riemann problem (i.e. the decay of a step function into waves) at the cell interface and compute the fluxes directly from the wave propagation. Accurate for gas-dynamics, but difficulties in MHD: degenerate eigenvector because of $\nabla \cdot \mathbf{B} = 0$.

MHD numerics (continued)

- Error terms:

$$
\begin{aligned}
\Delta x \frac{\partial U}{\partial t} &= -(f_{i+\frac{1}{2}} - f_{i-\frac{1}{2}}) \\
&+ a_1 (\Delta x)^2 \tfrac{\partial^2}{\partial x^2} F(U) + b_1 (\Delta x)^3 \tfrac{\partial^3}{\partial x^3} F(U) \\
&+ a_2 (\Delta x)^4 \tfrac{\partial^4}{\partial x^4} F(U) + b_2 (\Delta x)^5 \tfrac{\partial^5}{\partial x^5} F(U) \\
&+ \ldots
\end{aligned}
$$

  $\Rightarrow$ error terms with even derivatives cause diffusion

  $\Rightarrow$ error terms with odd derivatives cause dispersion

  $\Rightarrow$ central difference schemes have no diffusion, but dispersion, big problem at shocks and discontinuities

  $\Rightarrow$ first order schemes are less dispersive, but very diffusive

  $\Rightarrow$ see examples: wiggles at discontinuities

- Monotonicity: A scheme is called monotone if it lets no new extrema develop in the solution (that is exactly what we want).

MHD numerics (continued)

- Harten's (Di)Lemma:
  A monotone scheme is at most first order accurate!
  Thus, a globally monotone scheme will always be very diffusive.

- Solution to Harten's (Di)Lemma:
  Hybridize the numerical fluxes: Use first order numerical flux (f^l) where a new extremum might develop (like at a shock), and use high order fluxes (f^h) where the solution is smooth.

$$f_{i+\frac{1}{2}} = \theta_{i+\frac{1}{2}} f^h_{i+\frac{1}{2}} + (1 - \theta_{i+\frac{1}{2}}) f^l_{i+\frac{1}{2}}$$

  $\Rightarrow$ Obviously, $0 \leq \theta \leq 1$. θ can be a function of anything, but generally depends on gradients of the solution.

- The switch function θ is called *Flux Limiter*. There is no optimal flux limiter. A few choices:

  $\Rightarrow$ Hartens “edge condition” flux limiter

  $\Rightarrow$ vanLeer's flux limiter

  $\Rightarrow$ Flux Corrected Transport (FCT)

  $\Rightarrow$ Total Variance Diminishing (TVD) schemes (the monotonicity constraint is somewhat relaxed)

  $\Rightarrow$ ...

MHD numerics (continued)

- Keeping divergence of $\mathbf{B}$ zero

$\Rightarrow$ $\nabla \cdot \mathbf{B} = 0$ is an initial condition, $\nabla \cdot \mathbf{B}$ is conserved by Faraday's law:

$$\nabla \cdot \frac{\partial \mathbf{B}}{\partial t} = \frac{\partial (\nabla \cdot \mathbf{B})}{\partial t} = -\nabla \cdot \nabla \times \mathbf{E} = 0$$

$\Rightarrow$ $\nabla \cdot \mathbf{B}$ cleaning, projection method: solve

$$\nabla^2 \Psi = -(\nabla \cdot \mathbf{B})$$

and correct the field:

$$\mathbf{B}' = \mathbf{B} + \nabla \Psi$$

requires the numerical solution of a Poisson equation (expensive). Can only be as good as the solution of Ψ.

$\Rightarrow$ $\nabla \cdot \mathbf{B}$ convection: Effectively modify equations so that $\nabla \cdot \mathbf{B}$ convects through the system (Gombosi):

$$\frac{d(\nabla \cdot \mathbf{B})}{dt} = 0$$

$\nabla \cdot \mathbf{B}$ must convect out of the system (inner magnetosphere?).

$\Rightarrow$ Use numerical $\nabla \cdot$ and $\nabla \times$ operators with $\nabla \cdot \nabla \times \mathbf{\Psi} = 0$

$\Rightarrow$ Use a magnetic flux conservative scheme that keeps $\nabla \cdot \mathbf{B} = 0$ to roundoff error.

MHD numerics (continued)

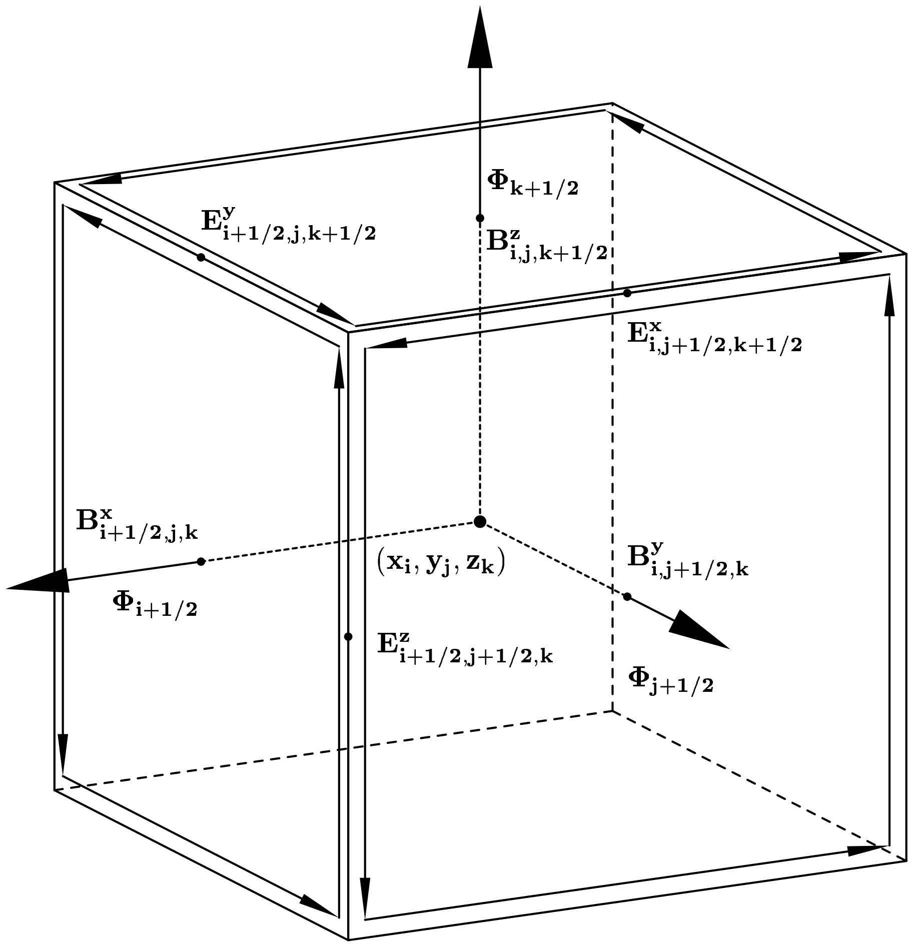
⇒ magnetic flux conservative scheme (continued)
place the magnetic field components on the center of cell faces:

$$(B_x)_{i+\frac{1}{2},j,k},\ \ (B_y)_{i,j+\frac{1}{2},k},\ \ (B_z)_{i,j,k+\frac{1}{2}}$$

⇒ and the electric field (a numerical flux) on the centers of the cell edges:

$$(E_x)_{i,j+\frac{1}{2},k+\frac{1}{2}},\ \ (E_y)_{i+\frac{1}{2},j,k+\frac{1}{2}},\ \ (E_z)_{i+\frac{1}{2},j+\frac{1}{2},k}$$

[*Evans and Hawley, 1987*].



MHD numerics (continued)

⇒ magnetic flux conservative scheme (continued) then:

$$\frac{\partial}{\partial t}(B_x)_{i+\frac{1}{2},j,k} =$$

$$\{(E_y)_{i+\frac{1}{2},j,k+\frac{1}{2}} - (E_y)_{i+\frac{1}{2},j,k-\frac{1}{2}}\}/\Delta z$$

$$-\{(E_z)_{i+\frac{1}{2},j+\frac{1}{2},k} - (E_z)_{i-\frac{1}{2},j+\frac{1}{2},k}\}/\Delta y$$

⇒ analogous for B_y and B_z

⇒ By advancing the field components in this way on all 6 cell faces and summing up it follows:

$$\frac{\partial}{\partial t}\int\int_{cell} \Phi df =$$

$$= \Delta y \Delta z (\frac{\partial B_x}{\partial t})_{i-\frac{1}{2}} + \Delta y \Delta z (\frac{\partial B_x}{\partial t})_{i+\frac{1}{2}} + \Delta x \Delta z (\frac{\partial B_y}{\partial t})_{j+\frac{1}{2}} + \ldots$$

$$= \{((E_y)_{i+\frac{1}{2},j,k+\frac{1}{2}} - (E_y)_{i+\frac{1}{2},j,k+\frac{1}{2}})+$$

$$((E_y)_{i+\frac{1}{2},j,k-\frac{1}{2}} - (E_y)_{i+\frac{1}{2},j,k-\frac{1}{2}}) + \ldots\}\Delta x \Delta y \Delta z$$

$$= 0$$

thus Φ = const.

⇒ The field can be initialized divergence free by using a vector potential $\mathbf{A}$ in place of $\mathbf{E}$.

MHD numerics (continued)

- How to handle stretched grids:

  $\Rightarrow$ grid coordinates given by analytic function: $x(i,j,k)$, $y(i,j,k)$, $z(i,j,k)$, then:

$$\frac{\partial}{\partial x}F(x,y,z) =$$

$$= \frac{\partial F}{\partial i}\frac{\partial i}{\partial x} + \frac{\partial F}{\partial j}\frac{\partial j}{\partial x} + \frac{\partial F}{\partial k}\frac{\partial k}{\partial x}$$

$$= \frac{\partial F}{\partial i}\Big(\frac{\partial i}{\partial x}\Big)^{-1} + \frac{\partial F}{\partial j}\Big(\frac{\partial j}{\partial x}\Big)^{-1} + \frac{\partial F}{\partial k}\Big(\frac{\partial k}{\partial x}\Big)^{-1}$$

  $\Rightarrow$ analogous for y and z derivative.

  $\Rightarrow$ particularly simple for stretched grid:

$$\frac{\partial}{\partial x}F(x,y,z) = \frac{\partial F}{\partial i}\Big(\frac{\partial i}{\partial x}\Big)^{-1}$$

$$\frac{\partial}{\partial y}F(x,y,z) = \frac{\partial F}{\partial j}\Big(\frac{\partial j}{\partial y}\Big)^{-1}$$

$$\frac{\partial}{\partial z}F(x,y,z) = \frac{\partial F}{\partial k}\Big(\frac{\partial k}{\partial z}\Big)^{-1}$$

Validation

- Compare with non-trivial solutions of the MHD equations (few available):

  $\Rightarrow$ shock tube problems (check for RH conditions, expansion shocks, flux limiters). Table of shock tube parameters used by ISTP:

ρ_1	ρ_2	p_1	p_2	v_{x1}	v_{x2}	v_{y1}	v_{y2}	v_{z1}	v_{z2}	B_{x1}	B_{x2}	B_{y1}	B_{y2}	B_{z1}	B_{z2}	γ
1	0.125	1	0.1	0	0	0	0	0	0	0	0	0	0	0	0	1.4
1	0.125	100	0.1	0	0	0	0	0	0	0	0	0	0	0	0	1.4
1	0.125	1	0.1	0	0	0	0	0	0	0.75	0.75	0	0	1	-1	2
1	0.125	1	0.1	0	0	0	0	0	0	0	0	10	0.5	10	0.5	2
1	0.125	100	0.1	0	0	0	0	0	0	0.75	0.75	0	0	1	-1	2
1	0.125	100	0.1	0	0	0	0	0	0	0	0	10	0.5	10	0.5	2
1	0.125	1	0.1	0	0	0	0	0	0	0	0	0	0	0.1	0.1	2

  $\Rightarrow$ 2D/3D self-similar solutions by Low [1982, Ap. J.].

- $\mathbf{B}$ convection test to evaluate $\nabla\cdot\mathbf{B}=0$: initialize a non-trivial $\mathbf{B}$ field from $\nabla\times\mathbf{A}$ such that $\mathbf{B}=0$ in most of the domain (magnetic bubble) and convect with $\mathbf{V}$=(V_x,0,0). $\nabla\cdot\mathbf{B}=0$ violation will become evident by B_x sticking to the grid:

$$\frac{\partial\mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}$$

$$\Rightarrow\qquad\frac{d\mathbf{B}}{dt}=\mathbf{V}(\nabla\cdot\mathbf{B})$$

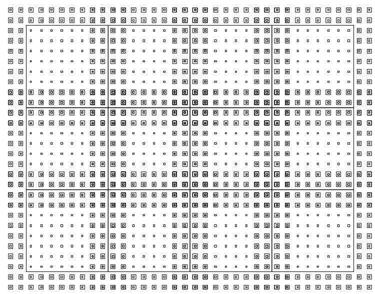
- Convergence test: run same problem twice: with highest resolution possible and with factor two lower resolution. The results should be qualitatively the same and quantitatively similar.

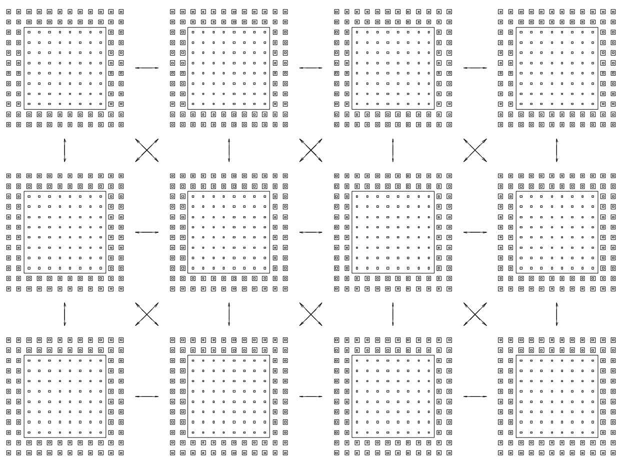
- Reality test: drive model with observed solar wind and IMF and compare results with *in situ* observations.

Computer issues

- Parallel implementation on MIMD (Multiple Instruction - Multiple Data) computers:

  ⇒ domain decomposition, guard cells, message passing:





  ⇒ extra nodes for: ionosphere(1), I/O(1), restart file manager(1), and for things to come

Computer issues (continued)

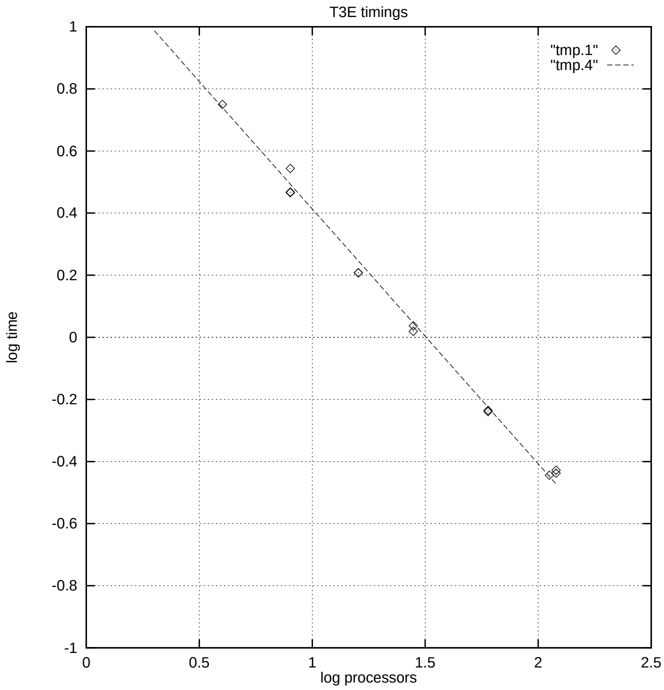
⇒ runs on IBM/SP2, CRAY/T3E, workstation (Beowulf) cluster, ...

⇒ language: f77, macro preprocessor, spag, ftnchek.

⇒ f90/HPF not mature enough and/or generally available, would require rewriting the code.

⇒ using MPI message passing library, either native or MPICH.

⇒ excellent scalability:



⇒ computing chore: $\sim$ 2000 Flop/gridpoint/timestep.

⇒ current machines: $\sim$ 100 MFlop/s.

⇒ realtime: 60 nodes, 10^6 cells.

⇒ output: several GB per run.

Things to do, given unlimited time and resources

- Adding more physics: inner magnetosphere, ring current

⇒ bounce aqveraged drift equations:

$$
\begin{aligned}
\frac{d\lambda_k}{dt} = (\frac{\partial}{\partial t} + \mathbf{v}_k \cdot \nabla)\lambda_k = & \quad 0 \\
\frac{d\eta_k}{dt} = (\frac{\partial}{\partial t} + \mathbf{v}_k \cdot \nabla)\eta_k = & \quad 0 \\
\mathbf{v}_k = & \quad B^{-2} q_k^{-1} (q_k \mathbf{E} \times \mathbf{B} + \lambda_k \mathbf{B} \times \nabla V^{-2/3}) \\
\lambda_k = & \quad W_k V^{2/3} \\
\eta_k = & \quad n_k V \\
V = & \quad \int ds/B
\end{aligned}
$$

[*Harel et al., 1981; Wolf et al.,1983; 1991; 1993*]

⇒ magnetic field: dipole, ...

⇒ plasma represented by macroparticles of constant λ_k and η_k

⇒ 32 log spaced energy levels, 0.1 to 300 keV

⇒ currently one species: protons

⇒ plasma loss due to pitch angle scattering: 10% of strong pitch angle scattering limit for protons

⇒ plasma loss due to charge exchange with exospheric hydrogen:[*Smith and Bewtra, 1978; Anderson et al., 1987*]

Things to do ..... (continued)

- Inner magnetosphere (continued)

  $\Rightarrow$ plasma loss due to Coulomb drag with plasmaspheric electrons: [*Fok et al., 1991*]

  $\Rightarrow$ 10° by 0.1 R_E grid in the magnetic equator, 1.2 $\leq$ L $\leq$ 7

  $\Rightarrow$ 0.1 to 0.5 million macroparticles per species, 1 processor

  $\Rightarrow$ long timesteps ($\approx$1 minute)

Things to do ..... (continued)

- Coupling with thermosphere - ionosphere circulation models (CTIM, TIECGM, ...)

  $\Rightarrow$ need: ionospheric parameters ($j_\parallel$, precipitation)

  $\Rightarrow$ provide: better ionospheric conductances, potential (neutral wind coupling, flywheel effect), ionospheric outflow.

- Resolution, resolution, resolution

  $\Rightarrow$ basic scaling law: T $\sim$ N^4 $= h^{-4}$, but computer power grows exponentially with time: