

## Plasmoid Formation in Current Sheet with Finite Normal Magnetic Component

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Current sheet configurations in natural and laboratory plasmas are often accompanied by a finite normal magnetic component that is known to stabilize the two-dimensional resistive tearing instability in the high Lundquist number regime. Recent magnetohydrodynamic simulations indicate that the nonlinear development of ballooning instability is able to induce the formation of  $X$  lines and plasmoids in a generalized Harris sheet with a finite normal magnetic component in the high Lundquist number regime where the linear two-dimensional resistive tearing mode is stable.

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Plasmoid often refers to a finite two-dimensional (2D) region of closed magnetic flux bounded by a separatrix with a single  $X$  point [1,2]. An isolated magnetic island in the downstream region of a Sweet-Parker current sheet is also generally called a plasmoid (e.g., [2]). Plasmoids are often found in natural and laboratory plasmas in association with various eruptive processes, such as those observed in solar corona, magnetosphere, and magnetic fusion experiments. Plasmoid formation has been believed to be the origin of substorm onset [3], and recently it has received renewed interest due to its potential roles in the universal process of fast reconnection [4]. Characteristically, a plasmoid could spontaneously form in the current sheet region with a finite magnetic field component  $B_n$  normal to the neutral sheet plane. However, it has not always been clear how a plasmoid would spontaneously form without an external driver in such a current sheet configuration where no  $X$  line preexists.

The formation of plasmoids has been mostly investigated for the Earth's magnetotail configuration in the context of the substorm onset problem [1–3,5–8]. In those studies the weakly 2D current sheet with finite  $B_n$  is used to model the initial static equilibrium of near-Earth magnetotail plasma. The 2D configuration becomes unstable to two-dimensional tearing type of perturbations when the plasma resistivity is sufficiently large. In its nonlinear stage, the unstable 2D resistive mode alone, which has been referred to by many as the “2D tearing instability” (e.g., [9–17]) and recently as the “axial tail instability” [18,19] in the context of magnetotail plasma, can induce the formation of the  $X$  line and plasmoid. In reality, however, magnetotail current sheets are often in regimes where the effective plasma resistivity might be too weak for the onset of 2D resistive tearing instability due to the strong stabilization from  $B_n$  (e.g., [15]). It has remained an interesting question how plasmoids would spontaneously form in the weakly resistive current sheets where no  $X$  line preexists and the finite  $B_n$  is sufficient to stabilize 2D resistive modes.

Our recent three-dimensional (3D) magnetohydrodynamic (MHD) simulations of the plasmoid formation process in the current sheet with finite  $B_n$  and weak resistivity have shown a significant difference from 2D simulations due to the 3D effects. In particular, the inclusion of the spatial variation in the equilibrium current direction (which is the  $y$  direction in the Cartesian coordinates defined later) allows the presence of ballooning instability (e.g., [20–22]), which has demonstrated its critical roles in the plasmoid formation process in the higher Lundquist number regimes where the linear 2D resistive modes of the current sheet are stable. In those regimes the thin current sheet with finite  $B_n$  is susceptible to finite- $k_y$  ballooning instability whose growth time scale is sub-Alfvénic. Here  $k_y$  is the wave number in the  $y$  direction. The nonlinear ballooning growth tends to stretch the current sheet and reduce  $B_n$ . As a consequence, magnetic  $X$  points appear and plasmoids start to form. All  $k_y$  components, including the  $k_y = 0$  component, contribute to the nonlinear ballooning growth that leads to the formation of plasmoids. These simulation results suggest a new mechanism for plasmoid formation in the current sheet with finite  $B_n$  in the high Lundquist number regimes. We briefly report and discuss these findings in this Letter.

We consider a generalized Harris sheet configuration in Cartesian coordinates  $(x, y, z)$  where  $\mathbf{B}_0(x, z) = \mathbf{e}_y \times \nabla\Psi(x, z)$ ,  $\Psi(x, z) = -\lambda \ln(\cosh[F(x)(z/\lambda)]/F(x))$ , and  $\ln F(x) = -\int B_{0z}(x, 0)dx/\lambda$ . Here  $\lambda$  is the current sheet width,  $\mathbf{e}_y$  is the unit vector in the  $y$  direction, and all other symbols are conventional. The profile of  $B_n = B_{0z}(x, 0)$  has a minimum region along the  $x$  axis (Fig. 1). Such a configuration was previously used to model the near-Earth magnetotail [19,23]. Unlike the conventional Harris sheet, where  $B_n$  and the magnetic curvature are zero everywhere, the generalized Harris sheet equilibrium shown in Fig. 1 has regions of unfavorable magnetic curvature mostly around  $z = 0$  due to the presence of finite  $B_n$ . Hence the generalized Harris sheet is susceptible to ballooning instability. Global simulations have identified signatures of both

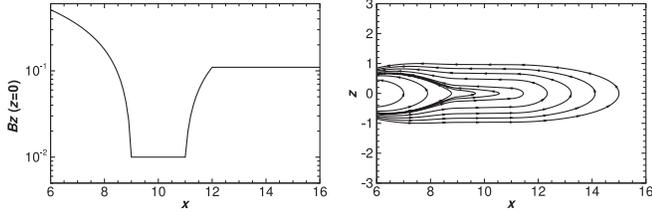


FIG. 1.  $B_{0z}(x, 0)$  profile (left) and magnetic field lines (right) for the generalized Harris sheet equilibrium considered in this Letter.

ballooning instability and axial tail instability near the minimum  $B_n$  region along the  $x$  axis [18,24], and recent MHD analysis indicates that such a configuration is indeed unstable to the axial tail instability but only in the low Lundquist number regime ( $S \lesssim 10^3$ ) [19].

To further investigate the stability of the configuration in a higher Lundquist number regime, a full set of resistive MHD equations are solved in the 3D domain as an initial-boundary value problem:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \mu \nabla \cdot (\rho \mathbf{w}), \quad (2)$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{u}, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad (4)$$

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J}, \quad (5)$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \quad (6)$$

where  $\rho$  is the mass density,  $\mathbf{u}$  is the plasma flow velocity,  $p$  is the pressure,  $\mathbf{E}$  is the electric field,  $\mathbf{B}$  is the magnetic field,  $\mathbf{J}$  is the current density, the adiabatic index  $\gamma = 5/3$ , and  $\mathbf{w} = \nabla \mathbf{u} + (\nabla \mathbf{u})^T - (2/3)\mathbf{I} \nabla \cdot \mathbf{u}$ . In a weakly collisional or collisionless plasma, both resistivity  $\eta$  and viscosity  $\mu$  are small in the absence of anomalous sources. The above set of equations have been implemented in both the linearized and the fully nonlinear version in the NIMROD code [25] used in our computation. A solid, no-slip wall boundary condition has been imposed on the sides of the computation domain in both the  $x$  and  $z$  directions, so that any potential influence from an external inward flow can be excluded. The boundary condition in the  $y$  direction is periodic. The spatial and temporal variables are normalized with the equilibrium scale length (e.g., Earth radius) and the Alfvénic time  $\tau_A$ , respectively.

Linear calculation indicates that the current sheet configuration shown in Fig. 1 is unstable to the 2D resistive tearing or axial tail instability ( $k_y = 0$ ) in the lower

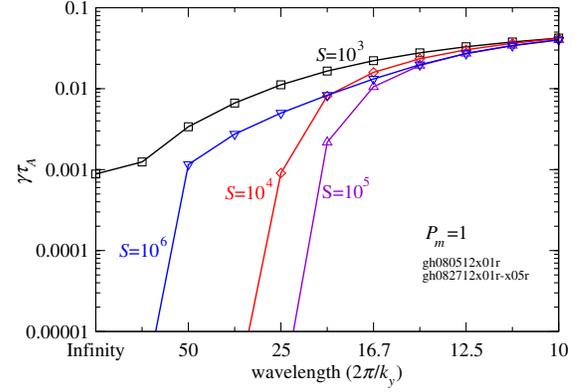


FIG. 2 (color online). Linear growth rate as a function of the wavelength in the  $y$  direction for different Lundquist number ( $S$ ) regimes. The magnetic Prandtl number  $P_m \equiv \mu/\eta = 1$  for all cases.

Lundquist number regime ( $S \lesssim 10^3$ ). The inclusion of spatial variation in the  $y$  direction significantly enhances the linear growth, particularly in the higher  $S$  regime when the 2D resistive tearing or axial tail mode is stable (Fig. 2). The enhanced linear growth of the finite- $k_y$  instability remains effective and becomes more relevant in the more realistic collisionality regime ( $S \gtrsim 10^6$ ), thus making the instability a viable mechanism for explaining the faster sub-Alfvénic time scale of the current sheet evolution in situations where the sources for large anomalous resistivity are not available.

We now consider the nonlinear plasmoid formation process in the same current sheet configuration in a less resistive regime  $S = 10^4$ , where the 2D resistive tearing mode is linearly stable and a plasmoid cannot spontaneously form internally from a purely 2D process ( $k_y = 0$ ). However, the inclusion of the 3D effects leads to an entirely new scenario where the plasmoid formation can be nonlinearly driven by a finite- $k_y$  ballooning instability. To demonstrate such a scenario, we report results from a representative numerical case where the simulation is

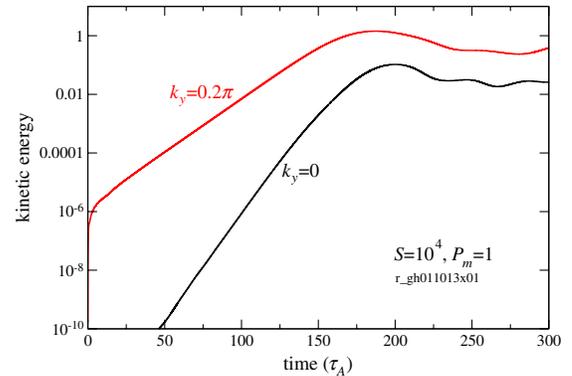


FIG. 3 (color online). Kinetic energy growth of the  $k_y = 0$  and  $k_y = 0.2\pi$  components of the nonlinear perturbation.

initialized with a small magnetic perturbation whose magnitude is about one-tenth of the minimum  $B_n$ . The initial perturbation is monochromatic in the  $y$  direction with a wavelength of 10. A finite element mesh of  $64 \times 64$  with a polynomial degree of 5 in each direction is used for the  $x$ - $z$  domain. In the  $y$  direction, 32 Fourier collocation points are used to resolve Fourier components in the range of  $0 \leq k_y L_y / 2\pi \leq 10$ , where  $L_y = 100$  is the domain size in  $y$ . The perturbation quickly settles into a linearly growing ballooning instability first and subsequently drives the growth of the  $k_y = 0$  component through nonlinear coupling (Fig. 3). The entire nonlinear evolution is dominated by the high  $k_y$  ( $k_y L_y / 2\pi = 10$ ) component. A natural consequence of the nonlinear ballooning drive is the formation of plasmoids within the  $x$ - $z$  plane.

To illustrate the plasmoid formation process, we track the evolutions of the pressure contour in the  $z = 0$  plane and the magnetic field lines crossing a set of fixed points along an  $x$  axis ( $y = -90, z = 0$ ) (Fig. 4). The first stage of nonlinear evolution, as represented by the plot at  $t = 180$  (the upper left panel in Fig. 4), is dominated by the growing ballooning finger structures in the  $z = 0$  plane extending in the  $x$  direction. The magnetic field lines are mostly frozen-in to the plasma and they move along with the extending fingers, which results in a stretching and thinning of the current sheet. The reduction of the normal component  $B_n$  in the  $z = 0$  plane appears to be the most in extent near the moving fronts of the extending fingers, as evidenced by the formation of a plasmoid in one of those locations around  $x = 13.5$  at  $t = 190$  (the upper right panel in Fig. 4). The

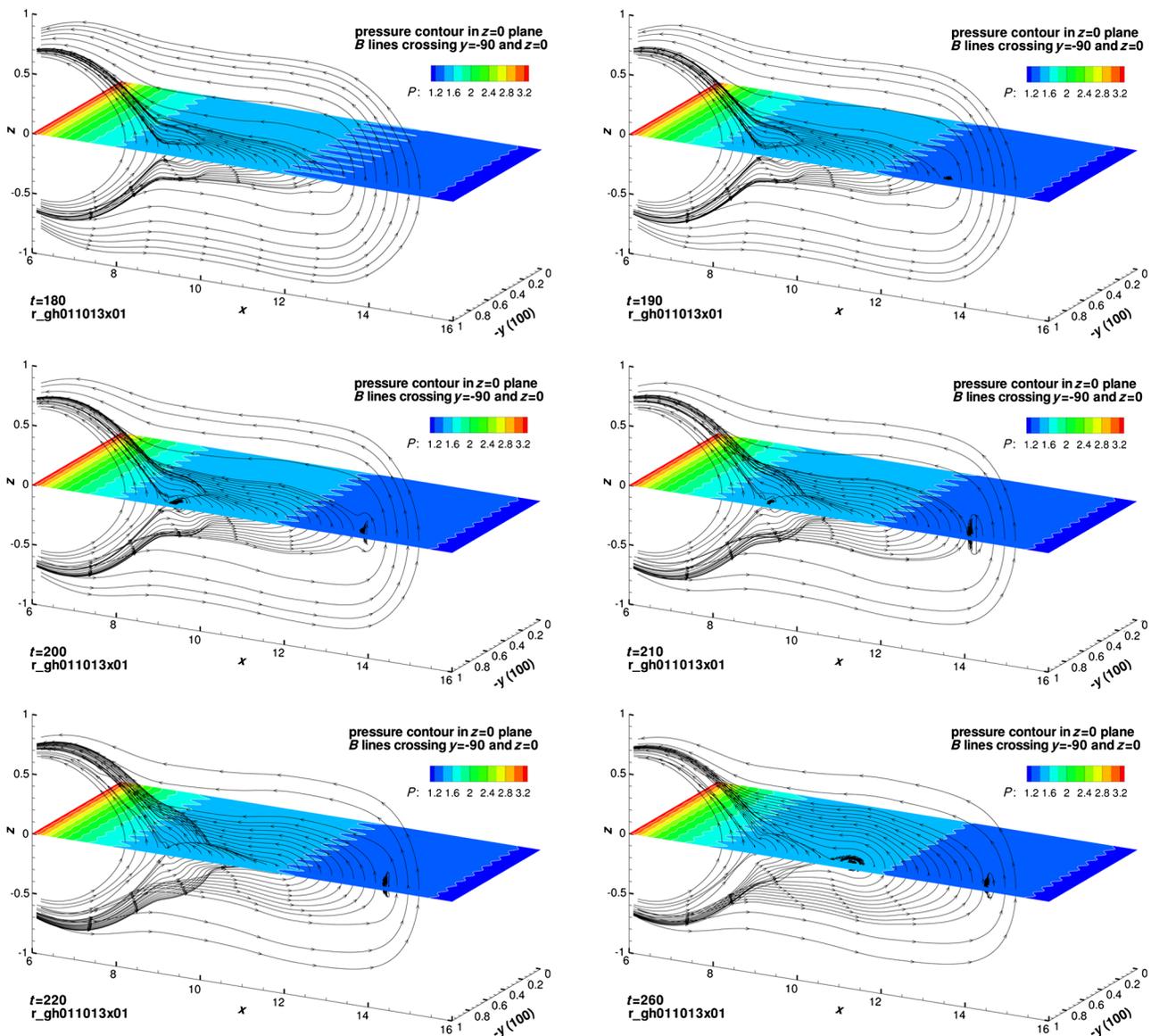


FIG. 4 (color online). Total pressure contours in the  $z = 0$  plane and magnetic field lines crossing the  $x$  axis at  $y = -90, z = 0$  at selected times ( $t = 180, 190, 200, 210, 220, 260$ ).

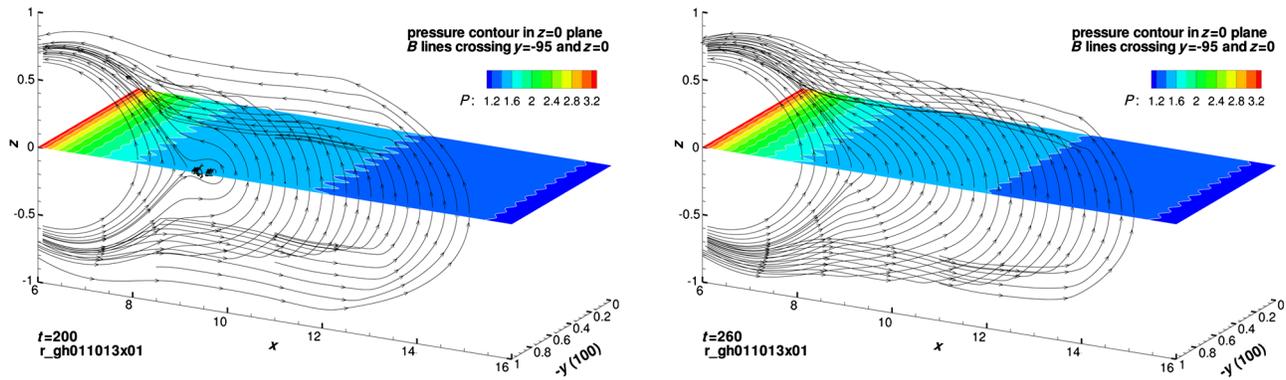


FIG. 5 (color online). Total pressure contours in the  $z = 0$  plane and magnetic field lines crossing the  $x$  axis at  $y = -95$ ,  $z = 0$  at selected times ( $t = 200, 260$ ).

plasmoid continues to grow in size and move in the positive  $x$  direction, even when the finger length in pressure contour has started to decrease as the amplitude of the ballooning instability has reached saturation since the time  $t \approx 190$ .

In addition to the formation of a plasmoid in close association with the extruding fronts of nonlinear ballooning fingers, other plasmoids have also formed in the wakes of those ballooning finger fronts (Fig. 4). Almost in parallel to the course of the plasmoid formation described in the previous paragraph, a second plasmoid starts to form since the beginning of the nonlinear ballooning saturation phase around  $t = 190$  and becomes visible by  $t = 200$  to  $t = 210$  on those magnetic field lines crossing the  $z = 0$  plane in the region around  $x \approx 9.5$  (the two middle row panels in Fig. 4). This second plasmoid, however, appears to be rather transient. When  $t = 220$ , the plasmoid located near  $x \approx 9.5$  disappears along with a dipolarization of the magnetic field in that region (the lower left panel in Fig. 4). From that time, the field lines crossing the  $z = 0$  plane in the  $x \approx 11$  region have started to stretch in the positive  $x$  direction, which eventually leads to the formation of a third plasmoid in that region by the time  $t = 260$  (the lower right panel in Fig. 4).

Unlike in 2D simulations, the above 3D plasmoid formation process is different for different locations along the  $y$  direction. For example, for a different set of field lines crossing the  $x$  axis at  $y = -95$ ,  $z = 0$ , there is no plasmoid structure at  $t = 260$  (the right panel in Fig. 5). Similarly, at an earlier time  $t = 200$ , the plasmoid associated with the ballooning finger front at  $x = 14$  on the  $y = -90$ ,  $z = 0$  axis (as shown in the middle left panel in Fig. 4) does not exist on these field lines crossing the  $y = -95$ ,  $z = 0$  axis; only near  $x = 9.5$  a plasmoid structure remains with a slightly different shape (the left panel in Fig. 5). The variation of the plasmoid presence and appearance in the  $y$  direction strongly indicates that the plasmoid formation reported here is an intrinsically 3D process that is qualitatively different from the 2D process.

In summary, we demonstrated in simulations that nonlinear ballooning instability can effectively enable the

formation of plasmoids in a current sheet with a finite normal component in the higher Lundquist number regime where the 2D resistive tearing or axial tail mode is stabilized by the finite  $B_n$ . Our results are not limited to the specific current sheet model shown in Fig. 1 or the particular numerical settings. The scenario obtained here persists in our simulations based on the more realistic current sheet profiles that are continuous at any differential order and in simulations with higher resolutions as well as nonmonochromatic initial perturbations. We plan to report those additional simulation results elsewhere. Recent 2D and 3D kinetic simulations have also found that plasmoids can form in magnetotail configurations and regimes where the 2D resistive tearing mode itself would be stable [26–28]. The quantification of the full range of configuration and parameter space for the reported plasmoid formation mechanism and the comparison between the MHD and kinetic simulation results will be subjects of future studies.

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