Numerical Modeling of Space Plasma Flows: ASTRONUM-2011
ASP Conference Series, Vol. 459
N. V. Pogorelov, J. A. Font, E. Audit, and G. P. Zank, eds.

Algorithm to Find Quasi Two-dimensional Reconnection Topology in Three-dimensional Simulations

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Abstract. We present an algorithm to find topological x-points and x-lines in the gridded data from plasma simulations. The algorithm is different from algorithms that find topological nulls, and also finds x-line topology when a guide field is present. We present an example of an OpenGGCM simulation of magnetic reconnection at Earth's magnetosphere, and we discuss the dependence of the results on the free parameter present in the algorithm.

1. Introduction

Magnetic reconnection is a fundamental process in many naturally occurring plasmas that converts magnetic energy into mechanical energy (flow) and heat (Priest & Forbes 2000). For example, magnetic reconnection is thought to provide the energy that drives solar flares and Coronal Mass Ejections (CMEs); and in Earth's magnetoshere reconnection occurs between the Interplanetary Magnetic Field (IMF) and Earth's magnetic field at the magnetopause (Haerendel et al. 1978; Sonnerup et al. 1981) and between lobe field lines in the geomagnetic tail (Dungey 1961; Baker et al. 1996; Schindler & Birn 1999; Angelopoulos et al. 2008). Many models of reconnection assume a simple two-dimensional geometry which makes the analysis easier. However, in a realistic environment it is by no means clear whether a simple two-dimensional model can apply. It is, in fact, even difficult to define what reconnection is in a general three-dimensional setting. Various possible three-dimensional reconnection regimes have been discussed, for example, by Hesse et al. (2005) and Pontin (2011).

However, observations of reconnection at Earth's magnetopause (Øieroset et al. 1997; Phan et al. 2000), in the Earth's magnetotail (Hones et al. 1986; Hones 1979; Baker et al. 1996), and in the solar wind (Phan et al. 2006; Gosling 2007) often appear to be consistent with a simple two-dimensional picture of reconnection. In particular, multiple observations of the same reconnection line in the solar wind (Phan et al. 2006) are consistent with X-lines that are many of orders of magnitudes longer than their transverse sizes and thus can be described by simpler two-dimensional models. In the following we will call reconnection *quasi two-dimensional* if a local coordinate system can be found such that a topological X-line runs along one of its axes, and that the derivatives along that axis can be ignored because the scale size along that axis is larger than the relevant scales perpendicular to it. While quasi two-dimensional reconnection occurs in nature, it is not clear, however, whether it is ubiquitous or only occurs in certain settings.

If quasi two-dimensional occurs in nature, it should also occur in numerical models of, for example, the magnetosphere. In this paper we are mainly concerned about MHD models, however, the algorithm presented here should also work for other models, such as hybrid models or fully kinetic models. As an example to demonstrate the algorithm we use MHD simulations of the Earth's magnetosphere and its interaction with the solar wind using the Open Geospace General Circulation Model (OpenGGCM). The OpenGGCM is extensively documented in the literature (Raeder et al. 2001; Raeder 2003; Raeder et al. 2008) and the model is also available as a community model for runs on demand at the Community Coordinated Modeling Center (`ccmc.gsfc.nasa.gov`).

It is generally a difficult problem to find reconnection sites in three-dimensional simulations. Because a parallel electric field is a necessary condition for reconnection to occur (Hesse et al. 2005) one may search the simulation domain for large values of $E_{\|}$. However, large values of $E_{\|}$ are often caused by numerical diffusion, so care must be taken to include those numerical terms. In practice, this can be accomplished by calculating $E_{\|} = \mathbf{E}_{\text{push}} \cdot \mathbf{B}/B$, where $\mathbf{E}_{\text{push}}$ is exactly the electric field that enters Faraday's law, including all numerical terms that arise from the discretization and flux limiting, and $\mathbf{B}$ is the magnetic induction vector. Such an approach is possible, for example, in codes that use the curl of $\mathbf{E}_{\text{push}}$ to advance $\mathbf{B}$ in time such as codes based on the Constrained Transport (Evans & Hawley 1988) method. In other algorithms the true numerical diffusion may be hidden and difficult to extract. Even if regions of large $E_{\|}$ are found, they may not necessarily coincide with regions of reconnection although, at this point, this depends on how reconnection is defined. Other criteria, such as the existence of separators, may be required. More recently, Zenitani et al. (2011) proposed a scalar reconnection measure that can be used in place of $E_{\|}$, which is Lorentz invariant and thus more universal than $E_{\|}$ itself. If, on the other hand, it is *a priori* assumed that reconnection occurs in quasi two-dimensional fashion, a well defined magnetic topology exists that can be exploited. In the following, we describe the algorithm to find such sites, and we conclude by presenting an example.

2. Algorithm

While algorithms exist to find topological nulls in a gridded magnetic field (see Lau & Finn 1990, for example) it is more difficult to find and describe x-lines or separator surfaces. For the latter it is generally required to "paint" the grid with the topology of a field line passing through the grid points, i.e., to assign a value to each grid point depending on the topology of the field line passing through it. We call this value the topology number T, which is unique for each field line. For example, in case of the Earth's magnetosphere, each grid point that lies on a closed field line is assigned the topology number T=1. Similarly, each grid point that connects to the northern (southern) polar cap is assigned a value of T=2 (3), and each grid point that is not magnetically connected to Earth has a value of T=4. Similar classifications are possible, for example, in the solar corona, where field lines either connect to the sun, to interplanetary space, or both. Separator surfaces can then be visualized with reasonable accuracy by rendering iso-surfaces at intermediate T values. However, x-lines (also called "spines") are more difficult to find. In order to find points that mark an x-line we proceed as follows:

1. Scan the entire grid, or a selected region, for grid points such that itself and its immediate neighbors encompass all four possible topologies.

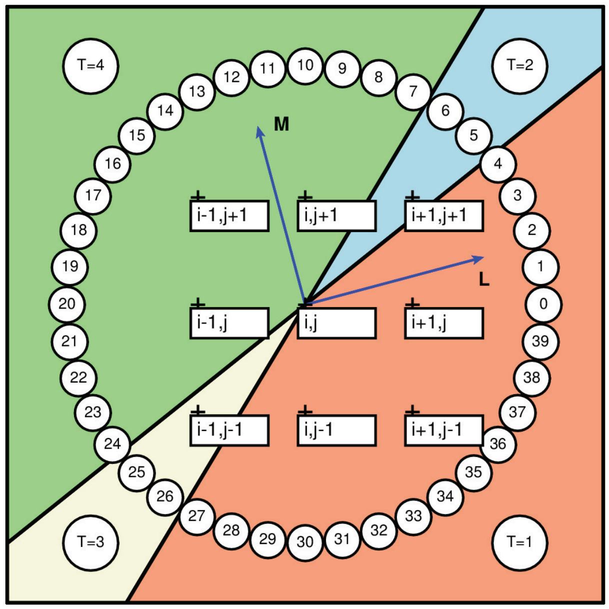

Figure 1. Schematic showing the magnetic field topology in the vicinity of a topological x-line. The grid point (i,j) is surrounded by grid points of all 4 possible topologies. There are a total of 27 points, $(i\text{-}1,j\text{-}1,k\text{-}1)$ to $(i{+}1,j{+}1,k{+}1)$ surrounding the x-line. A minimum variance analysis of the set $\{\mathbf{B}_{r,s,t}|r = i-1, i, i+1; s = j-1, j, j+1; t = k-1, k, k+1\}$ defines a local coordinate system through the eigenvectors $(\mathbf{L}, \mathbf{M}, \mathbf{N})$. $\mathbf{N}$ is the minimum variance direction, which is by definition locally parallel to the x-line. The intermediate ($\mathbf{M}$) and maximum ($\mathbf{L}$) variance directions span the plane shown here, but $\mathbf{L}$ and $\mathbf{M}$ have no specific relation to the reconnection process. Note, that the 9 gridpoints marked here do not need to lie in the $\mathbf{L}-\mathbf{M}$ plane.

2. For each grid point (i,j,k) so found, record the set of 27 magnetic field values $\{\mathbf{B}_{r,s,t}|$ $r{=}i\text{-}1$, i, $i{+}1$; $s{=}j\text{-}1$, j, $j{+}1$; $t{=}k\text{-}1$, k, $k{+}1\}$ associated with itself and its neighbors. If the magnetic topology is indeed that of quasi two-dimensional reconnection, then there should exist one direction in which the component of the magnetic field does not vary, i.e., the direction parallel to the x-line. To find that direction we employ the method of minimum variance (Sonnerup & Cahill 1967, 1968), which yields the local coordinate unit vectors $(\mathbf{L}, \mathbf{M}, \mathbf{N})$ that are the directions of maximum, intermediate, and minimum variance of the set $\{\mathbf{B}_{r,s,t}\}$. The vectors $\mathbf{L}$ and $\mathbf{M}$ span the plane of quasi two-dimensional reconnection, while $\mathbf{N}$ is parallel to the x-line.

3. We still need to assert that exactly 4 different topologies meet near (i,j,k). For that purpose we determine the magnetic topology of a set of points located on a circle about the point (i,j,k) lying in the $\mathbf{L}-\mathbf{M}$ plane, as depicted in Figure 1. The radius R of that circle is a free parameter. It is of the order of a few times the grid spacing and will be discussed further below. For each of the m points the magnetic topology is determined by tracing the field line through that point l and we record the topology number as $T(l)$. In the figure m=40 for illustration purposes, but in practice a larger value should be used to determine the topology more accurately and to catch narrow exhaust channels. The set $\{T\}$ is now circularly shifted such that $T(0)$=1 and $T(m\text{-}1)\neq 1$. For example, in

Figure 1, if orange corresponds to T=1, green to T=4, yellow to T=3, and blue to T=2, then $\{T\} = \{1, 1, 1, 1, 1, 2, 2, 4, 4, 4, 4, \ldots, 1, 1, 1\}$, and $\{T\}$ needs to be circularly shifted 13 places to the right (equivalent to rotate the numbering clockwise by 13 places) to yield $\{T\} = \{1, 1, 1, 1, 1, 1, \ldots, 3, 3, 3\}$. Next, we test if the values of $\{T\}$ correspond to an expected permutation. In case of the Earth and the classification defined above these permutations would be 1-2-4-3 or 1-3-4-2. For other MHD systems the expected permutations may be different. Obviously, for the example in Figure 1, the permutation is 1-2-4-3 and the point (i,j,k) lies within the distance R of a topological reconnection site. If $\{T\}$ does not match the expected permutation the point (i,j,k) is dismissed as a possible quasi two-dimensional reconnection site.

4. We may now rotate the **L** and **M** vectors about **N** such that they dissect, for example, the subsets $\{T|T(i)=1\}$ and $\{T|T(i)=2\}$. The **L** and **M** vectors may not be strictly perpendicular; however, one of them lies along the axis of the inflow region, and the other lies on the axis of the outflow region. Defining these axes is useful for a detailed analysis of the reconnection process, such as tests of the Walen relation.

Once a set of points with proper quasi two-dimensional topology is found one can either visualize the set of points to trace the x-line, or select a subset of the points and plot for each 4 representative field lines that show how the fields merge at that location. To select such lines, one can, for example, take the midpoints of the subsets $\{T|T(i) = t\}$, where $t \in \{1, 2, 3, 4\}$ is one of the four possible topologies, and visualize the field lines that run through these points.

3. Example

Here, we show a reconnection example from OpenGGCM simulations of the magnetosphere. In this simulation, the IMF is mainly in the east-west direction, i.e., the IMF B_y component dominates. Under such conditions reconnection is highly time-dependent, leading to the formation of flux transfer events (FTEs), i.e., magnetic flux ropes (Russell & Elphic 1979; Raeder 2006). The OpenGGCM simulations shown here are for an observed FTE event (May 7, 2004); a detailed analysis of this simulation and comparisons with data will be published elsewhere.

Figure 2 shows the application of our algorithm to this simulation. We present snapshots from two different times. The left column shows the results for 22:24 UT, and the right column shows the results for 22:30 UT, i.e., 6 minutes later. The three rows show three different applications of the algorithm with different values of R, which is the only free parameter besides m to choose. The choice of m is generally not critical, as long as m is large enough to resolve the magnetic structure in the vicinity of the x-line. Here, we choose m=64. Choosing a much larger value for m will do no harm, other than requiring more computer cycles. On the other hand, R affects the results. A lower limit is essentially given by the grid size h. If R is significantly smaller than h, the algorithm will fail to detect a proper x-line topology for many candidate points found in step 1) of the algorithm. If R is too large, there is a chance that the search circle crosses regions that have nothing to do with the local topology. Thus, in practice, R should be of the order of a few times the local grid size h, i.e., $R = Sh$. In the example shown in Figure 2 we choose S=1,3,5 from top to bottom.

For the smallest R (top row in Figure 2) the x-line topology is often not detected. A larger value will reveal more points that belong to the topological structure. In Figure 2, the second row shows the case of $R = 3h$. Comparing panels (A) and (B), and

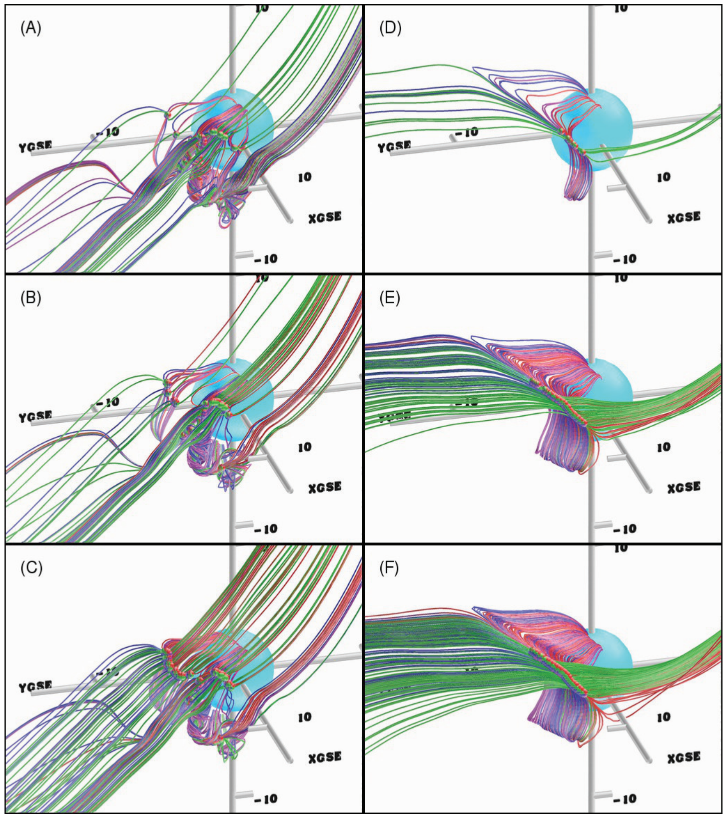

Figure 2. Rendering of the reconnection geometry, i. e., x-lines, using our algorithm. The left and right column are different times in the simulation. In each row a different radius R for the search circle is used, i.e., 1, 3, and 5 times the local grid spacing. The field lines are colored according to their topology. Green lines are unconnected (solar wind), blue lines connect to the southern hemisphere, red lines to the northern hemisphere, and purple lines are closed. We also draw a sphere at the root point of each line. These spheres then trace out the topological x-lines. The blue sphere about the center is the inner boundary of the MHD domain; it has a radius of 3.5 R_E.

panels (D) and (E), respectively, more details become evident with a larger search circle. Increasing the search circle further, i.e., $R = 5h$ in panels (C) and (F) may or may not reveal more detail. In panel (C), there are now 2 distinct and contiguous x-lines visible, which were not evident with smaller R (panels (A) and (B)). However, for the time shown at the right, using a larger circle does not reveal new information. Thus, in practice, some experimentation is required to find a suitable R. One also needs to make choices about the number of points and lines to plot. One can easily find too many points with x-line topology, such that, when all lines are plotted, the figure becomes to be too crowded to be useful.

Although this paper focuses on the algorithm and not on the physics, we note that the simulations show that (i) extended x-lines exist at the magnetopause, although they are often thought to be topologically unstable, and (ii) that FTEs are apparently associated with the presence of multiple x-lines, which was already found earlier for different IMF orientations (Raeder 2006).

4. Conclusions

We have shown that the brute force algorithm presented here is capable of finding magnetic x-points and x-lines in the gridded data of plasma simulations. The algorithm depends essentially on only one free parameter, i.e., the scale of the search circle R. We show that for suitable choices of R it is possible to find the quasi-2d loci of reconnection. This enables further analysis, such as of the extent and time evolution of x-lines, and the interplay between multiple x-lines. The algorithm also yields unit vectors along the inflow and the exhaust regions, which allows a more detailed analysis of the flows and fields in the vicinity of reconnection sites.

Acknowledgments. This work was supported by NSF grants OCI-0749125 and ATM-0639658. Computations were performed at the Texas Advanced Computing Center under Teragrid grant TG-MCA98N022.

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