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A SIMULATION STUDY OF MULTIPLE ION WAVE GENERATION DOWNSTREAM OF LOW MACH NUMBER QUASIPERPENDICULAR SHOCKS

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Abstract. The behavior of minor ions just downstream of a low Mach number quasiperpendicular shock is investigated both theoretically and by computer simulations. Because all ions see the same cross shock electric field their deceleration depends on their charge to mass ratio, yielding different downstream velocities. It is shown that these differences in velocity can lead to coherent wave structures in the downstream region of quasiperpendicular shocks with a narrow transition layer. These waves are shown to be multi ion hybrid waves in contrast to mirror waves and ion cyclotron waves. Under favorable conditions these waves should be observable both at interplanetary shocks and at planetary bowshocks.

Introduction

The study of shock waves in space plasmas has received considerable attention since the observation of planetary bow shocks and interplanetary shock waves. These observations initiated many theoretical studies and computer simulations, which brought a good insight into the nature of collisionless shocks. In particular perpendicular and quasiperpendicular shocks have been studied extensively and the basic generation mechanism for these shocks is well understood for a wide range of parameters [see e.g. Goodrich, 1985].

Downstream waves at quasiperpendicular shocks are known in single ion plasmas for high and low Mach numbers. Price et al. [1986] found that strong quasiperpendicular shocks lead to considerable ion temperature anisotropy downstream of the shock which can generate mirror waves and ion cyclotron waves. Temperature anisotropies have also been observed at low Mach number, low β perpendicular and quasiperpendicular shocks [Thomsen et al., 1985; Schopke et al., 1990]. However, wave amplitudes seem to be much lower in comparision to high Mach number shocks.

The presence of other ion species however leads to additional wave modes which should be observable at such low Mach number quasiperpendicular shocks. In an earlier paper Motschmann et al. [1991b] studied the velocity space filtering effect at shocks in a plasma consistent of multiple ion species. Deceleration of different ion species at the shock transition layer leads to a different bulk velocity of the respective ion species, even if one assumes that all species are incident on the shock with the same bulk speed. Such velocity filtering has been observed at interplanetary shocks for different solar wind minor ions [Ogilvie et al., 1982]. Although their results indicate velocity differences only for the component parallel to the field, the filtering mechanism should also be effective for the perpendicular component. More recently Fuselier et al. [1988] reported observations of shell like distributions of minor ions in the magnetosheath and interpreted their results also as a shock filtering effect due to the shock potential. It should be noted that both studies use data which are averaged over rather long time periods and are thus not showing any waves which may be related to the presence of minor ions.

The different velocities of the ion species cannot be sustained downstream when the magnetic field has a substantial component perpendicular to the flow. Zhao et al. [1991] have shown that for supercritical quasiperpendicular fast shocks the energy is converted into excess heating of the minor ions. However, for subcritical, low Mach number shocks the free energy available in the differential velocities is rather converted into multi ion waves as will be shown later in this paper. In contrast to microscopic velocity space instabilities, such as the ion cyclotron or the mirror instability, the wave excitation mechanism is macroscopic.

Multi ion waves were first studied by Buchsbaum [1960] and by Smith and Brice [1964] for a cold plasma consisting of different ion species. In analogy to the (electron–ion) hybrid modes these waves were named ion–ion hybrid modes. In the present paper we will investigate the excitation mechanism and the properties of such waves downstream of a quasiperpendicular low Mach number multi ion shock in more detail. For simplicity we consider a plasma in which the major species is H^+ and the minor species are He^{++} and He^+. The primary reason for this choice was the well separated mass to charge ratio for these ions. However, our results can as well be applied to plasmas with other minor ion species.

In the remainder of the paper we will first present a brief analytical analysis of the jump relations and dispersion properties of the ion–ion hybrid mode. We will then present the results of hybrid simulations and finally discuss these results and address the observability of the ion–ion hybrid mode. SI units are used throughout this paper.

Analytical Analysis

The modifications of the Rankine–Hugoniot relations in a multiple ion plasma were previously investigated by Motschmann et al. [1991a] where the single ion Rankine–Hugoniot relations were generalized to the multi ion case, which will be abbreviated MIRHR (Multiple Ion Rankine Hugoniot Relations) in the following. It was shown that the MIRHR are tractable for a low Mach number perpendicular shock in the bi ion case. These results are also applicable to plasmas with more than two ion species as long as the plasma consists of one major component and the density of the other components is small compared to the density of the major component. In this case the interaction between the minor components can be neglected and the interactions of each of the minor components with the major component is seperable.

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For each species the deceleration and thus the loss of kinetic energy at the shock transition can be related to the cross shock potential drop:

$$\frac{m_i}{2}(U_0^2 - U_{i,d}^2) = -q_i e \int_{upstream}^{downstream} E_x dx = q_i e \Phi \quad (1)$$

where U_0 is the upstream bulk velocity of all species, $U_{i,d}$ is the downstream bulk velocity of species i, m_i and q_i are the ion mass and charge number respectively, E_x is the electric field along the shock normal and e the unit charge. The actual shock potential is of no interest here. Combining equation (1) for the major plasma constituent (H^+, index p) and a minor heavier constituent (i.e. He^+ or He^{++}, index h) leads to the downstream bulk velocity of the minor constituent:

$$U_{h,d} = \sqrt{U_0^2 - q_h \frac{m_p}{m_h}(U_0^2 - U_{p,d}^2)} \quad (2)$$

The differential velocities $U_{h,d} - U_{p,d}$ immediately downstream of the shock transition may be regarded as a nontrivial boundary condition for the downstream flow. When the magnetic field is perpendicular to the flow, the electric field in the rest frame of either ion species does not vanish and the ions are forced into gyromotion. The dispersion of the resulting wave modes was first analyzed by Buchsbaum [1960] and Smith and Brice [1964]. Considering all plasma constituents (ions and electrons) the dispersion relation for waves propagating perpendicular to the magnetic field reads:

$$\frac{c^2k^2}{\omega^2} = \frac{(1 - \sum_\alpha \frac{\omega_\alpha{}^2}{\omega^2 - \Omega_\alpha{}^2})^2 - (\sum_\alpha \frac{\Omega_\alpha}{\omega}\frac{\omega_\alpha{}^2}{\omega^2 - \Omega_\alpha{}^2})^2}{1 - \sum_\alpha \frac{\omega_\alpha{}^2}{\omega^2 - \Omega_\alpha{}^2}} \quad (3)$$

Ω_α and ω_α are the cyclotron and plasma frequency of species α, respectively. In the limit of low frequencies ($\omega \ll \Omega_e, \omega_e$, where the index e stands for electrons) equation (3) may be simplified to:

$$\omega^4 - \omega^2(\Omega_{cf}{}^2 + a_h V_A{}^2 k^2) + a_h \Omega_{bi}{}^2 V_A{}^2 k^2 = 0 \quad (4)$$

where $V_A = B/\sqrt{\mu_0(m_p N_p + m_h N_h)}$ is the Alfvèn velocity and Ω_{cf} and Ω_{bi} are the cutoff and bi ion hybrid resonance frequencies given by:

$$\Omega_{cf} = \frac{N_h}{N_p + N_h}\Omega_p + \frac{N_p}{N_p + N_h}\Omega_h \quad (5)$$

$$\Omega_{bi} = \sqrt{\Omega_p \Omega_h}\sqrt{\frac{N_p\Omega_h + N_h\Omega_p}{N_p\Omega_p + N_h\Omega_h}} \quad (6)$$

The coefficient a_h may be called an abundance coefficient and is given by:

$$a_h = 1 + \frac{N_p N_h}{(N_p + N_h)^2}\left(\sqrt{\frac{m_p}{m_h}} - \sqrt{\frac{m_h}{m_p}}\right)^2 \quad (8)$$

The cutoff and bi ion hybrid resonance frequencies are related by $\Omega_{cf} = \sqrt{a_h}\Omega_{bi}$, thus a_h is a measure of the frequency gap between cutoff and resonance. Bi ion waves with frequencies falling in this gap cannot exist.

While the dispersion relation (4) describes the normal modes of the plasma downstream of the shock, the differences of the velocities of the different ion species at the shock transition act as a boundary condition for the waves. Since this boundary condition fixes the wave phase, the multi ion hybrid waves must propagate in the upstream direction at the downstream bulk flow velocity. In other words, the Doppler shifted waves must not propagate in the rest frame of the shock transition. Thus, out of the 4 possible modes given by (4) only those with a phase velocity of $V_{ph} = \frac{\omega}{k} = -U_{p,d}$ are admissable. By combining the condition $\omega - kU_{p,d} = 0$ with the dispersion relation (4) and assuming $N_h \ll N_p$ we can derive the wavelength $\lambda_h = 2\pi/k$ of the downstream multi ion waves:

$$\lambda_h = 2\pi \frac{U_{p,d}}{eB_d}\frac{m_h}{q_h} \quad (9)$$

Here B_d is the downstream magnetic field.

As will be shown later in the simulation results, the waves can readily be seen in the ion moments. Since the waves are electromagnetic, they can also be seen in the fields. In order to approximate the expected wave amplitudes in the downstream magnetic field we consider the variation of the y–component of the electron momentum equation:

$$\delta(U_e B + E_y) = 0 \quad (10)$$

where B is along the z–axis. With:

$$U_e = \frac{N_p}{N_p + N_h}U_p + \frac{N_h}{N_p + N_h}U_h \quad (11)$$

one finds:

$$\frac{\delta B}{B} \simeq -\frac{N_h}{N_p}\frac{\delta U_h}{U_p} \quad (12)$$

where all quantities in (10–12) are taken in the downstream region. Since δU_h is given by the MIRHR and $|\delta U_h/U_p| \lesssim 1$ is always close to one the field fluctuations are mainly determined by the heavy ion abundance N_h/N_p.

Simulations

For the simulation studies we use a one dimensional selfconsistent hybrid code with kinetic ions and fluid electrons which has been generalized to the multiple ion case. A constant resistivity η provides the necessary dissipation [Winske and Leroy, 1985]. The shock is initialized by the injection method, i.e. the particles and fields are initialized to satisfy the MIRHR of a shock which is stationary in the rest frame of the simulation box. The field and fluid quantities are fixed at the upstream boundary. At the downstream boundary we apply open boundary conditions, i.e. the derivative of all field and fluid quantities are set to zero and particles can leave the box. Particles are injected at the left side of the box according to the fixed inflow boundary conditions. We consider a plasma consisting of protons with the minor ions He^+ and He^{++}. The upstream particle density is 25 particles per cell and per species. All species enter the simulation box with the same bulk velocity. Distance is normalized to the proton inertia length c/ω_p, where c is the speed of light. Time is normalized to the inverse of the upstream proton gyrofrequency $\Omega_p = eB/m_p$. Particle densities are normalized to the upstream proton density, the field and velocity are normalized to

Table 1. Shock parameters

Parameter	Value
Alfvén Mach number	1.58
shock normal angle θ_{Bn}	85°
electron beta β_e	0.75
proton beta β_p	0.01
resistivity	$0.001/\Omega_p$
N_{He+}/N_p	0.01
N_{He++}/N_p	0.05

their respective upstream values. The length of the simulation box is [illegible] c/ω_p with a gridsize of 0.46 c/ω_p and we choose a timestep of [illegible] Ω_p^{-1}. The shock parameters are summarized in Table 1.

A stationary shock develops after a few proton gyroperiods and a stationary structure is reached in the entire simulation domain after about [illegible] Ω_p^{-1}.

Results are shown in Figure 1. We first analyze the velocity jumps of the different ion species. It can be seen that the different ion species attain different velocities just downstream of the shock transition. The immediate downstream velocity of the He^{++} ions is greater than the velocity of the protons and the He^{+} velocity is higher than the He^{++} velocity. Because the field is almost perpendicular to the flow, the difference in the flow velocities cannot be compensated and the ions begin to gyrate.

The simulation gives $U_{p,d}$=$0.74U_0$ for the mean downstream velocity of the protons, which is equal to the predicted jump by the single ion Rankine-Hugoniot relations . For the minor ions we find velocity jumps of $U_{He+,d}$=$0.94U_0$ and $U_{He++,d}$=$0.88U_0$, respectively. The gyration amplitude in velocity space is twice the difference between the immediate downstream velocity of the respective heavy ion species and the mean velocity. The mean velocity is in this case very close to the proton velocity because of the low relative densities of the heavier ions. We find $2|U_{He+,d} - U_{p,d}|$=$0.54U_{p,d}$ for He^{+} and $2|U_{He++,d} - U_{p,d}|$=$0.28U_{p,d}$ for He^{++}. Because the waves have large amplitudes in the velocity moments of the heavier ions they should be observable with ion spectrometers with sufficient mass and time resolution. From Figure 1 one also sees that the wave structures extend quite far downstream of the shock in a coherent manner. The coherency of the waves depends on the thickness of the shock ramp. If the thickness of the shock ramp becomes comparable to the Larmor radius of the heavy ions, the ions would no longer be phase bunched immediately downstream of the shock. In this case the ions would spread on a ring in velocity space with the center of the ring at the bulk velocity and the radius equal to the velocity differences $|U_{He++,d} - U_{p,d}|$ and $|U_{He+,d} - U_{p,d}|$, respectively. The essential mechanism for forming the downstream waves is therefore the fact that all minor ions leave the shock ramp at essentially the same phase angle. Only a thin shock (compared to the heavy ion Larmor radius) lets the ions gyrate within a narrow range of phases and generate coherent waves. Multi ion hybrid waves can therefore only be observed at shocks of low Mach number and low ion beta. Examining the ion moments at different times shows that the waves are indeed stationary in the rest frame of the shock. For the wavelengths we find from Figure 1 $\lambda_{He^+} \simeq 20c/\omega_p$ and $\lambda_{He^{++}} \simeq 10c/\omega_p$. Both characteristic wavelengths can also be seen in the downstream magnetic field. The relative amplitudes (peak to peak) are approximately 0.07 for the He^{+} mode and 0.03 for the He^{++} mode, respectively.

Comparing the simulation results quantitatively with theory we find that the velocity jumps are in excellent agreement. For the wavelengths equation (9) predicts $\lambda_{He^+} \simeq 14c/\omega_p$ and $\lambda_{He^{++}} \simeq 7c/\omega_p$, while for the relative magnetic field wave amplitudes we should get from (12) 0.005 and 0.02 respectively. The differences are about 30 percent for the wavelengths and about one order of magnitude for the field amplitudes. There may be multiple reasons for these discrepancies. First, our theoretical results are based on cold plasma theory, whereas the plasma has a finite temperature in the simulation. Second, the waves have large amplitudes and may well be in a nonlinear regime. The dispersion relation (3) on the other hand is only strictly valid in the limit of small amplitudes. These limitations of the theoretical analysis may account for the differences of the wavelenghts between theory and simulation. As for the estimate of the magnetic field amplitudes our theoretical arguments

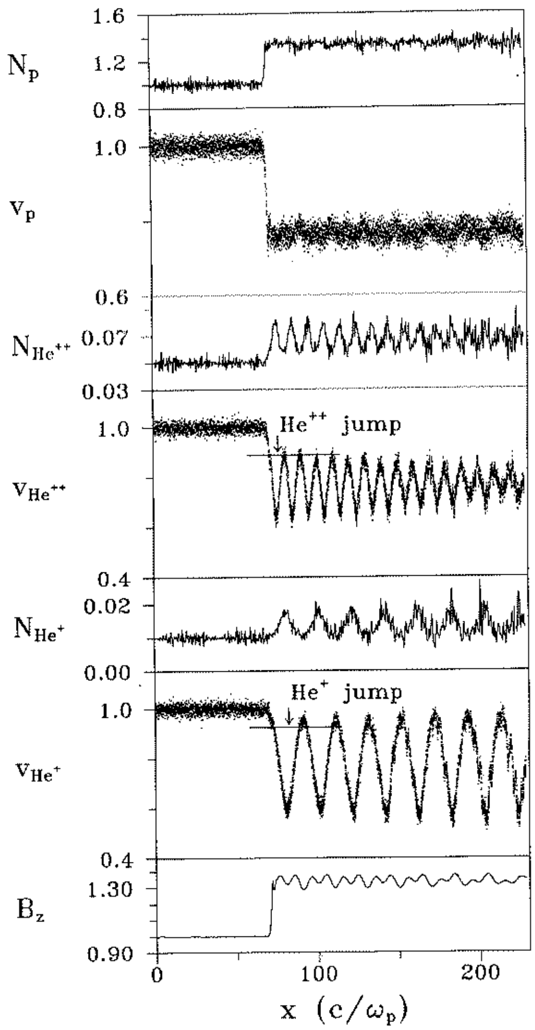

Fig. 1. Simulation results of a multiple ion, low Mach number quasiperpendicular shock and related downstream waves. From top to bottom: Proton density (N_p), Proton phase space (v_p), He^{++} density ($N_{He^{++}}$), He^{++} phase space ($v_{He^{++}}$), He^{+} density (N_{He^+}), He^{+} phase space (v_{He^+}), magnetic field (B_z).

(10–12) may be too simple for waves of such large amplitudes. Because the waves are possibly nonlinear we think that the simulation result is more applicable. Nevertheless, equations (9) and (12) might be useful in determining the basic wave properties.

Summary and Conclusions

We investigated the influence of single and double charged helium in a proton plasma on downstream waves at low Mach number quasiperpendicular shocks both theoretically and by means of computer simulations. We find that the MIRHR predict different downstream bulk velocities for the different ion species. Our simulation results are in good agreement with these findings. The different velocities can be explained by a filtering effect of the shock potential: all ions see the same potential, but due to their different mass to charge ratio they are decelerated to different velocities. The different ion velocities then lead to standing wave structures just downstream of the shock. Whether or not such standing structures can develop depends on the shock parameters. The shock dissipation region must be sufficiently thin, otherwise the phases of the ions become smeared out and no coherent wave structures can develop. A sufficient criterion is that the thickness of the dissipation layer is small compared to the heavy ion Larmor radius. These criteria are fulfilled for low ion beta, low Mach number quasiperpendicular shocks, as has been shown by our simulation results. Once stationary multi ion hybrid waves develop they can be seen most clearly in the ion moments (velocity and density), but also in the transverse magnetic field components. The dispersion properties of these waves have been derived and an estimate of the wavelengths and field amplitudes has been given, which are at least for the wavelengths in reasonable agreement with the simulation results.

To our knowledge there has not been any search for multi ion hybrid waves at interplanetary shocks or planetary bow shocks. However, the properties of these waves should make them observable for steady solar wind conditions. While the magnetic field fluctuations of the multi ion waves may be masked by ion cyclotron or mirror waves, the amplitudes of the velcities and the densities of the heavy ions are relatively large. This should make the multi ion waves observable with mass resolving ion spectrometers. For a typical solar wind plasma (B = 5 nT, U_0=400 km/sec) and a Mach 1.6 shock we find wavelengths of $\lambda_{He^+} \simeq 11000km$ and $\lambda_{He^{++}} \simeq 5500km$, which translate into periods of 275 sec and 138 sec respectively (assuming 40 km/sec relative velocity between a spacecraft and the shock). These wave periods fall into sampling rates of magnetometers and mass spectrometers.

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