PHYS 942 homework assignment #05

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PHYS 942
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Names ($\leq$3, write clearly): ______________________________

Due: Monday, November 14, 2016, at the lecture. **Show all your steps!**

1. (*30 points*) Jackson, problem 9.6. Integrate over t' first. Remember how we turned integrals over $\mathbf{J}$ into integrals over ρ. Beware of the retarded time when you differentiate.

2. (*30 points*) Jackson, problem 10.3. In part (a) "explain" means a detailed discussion of the assumptions that allow you to use the scalar potential (see Jackson section 5.9.B) approach. Write down the conditions under which the 'magnetostatic' assumption is valid. Then expand into a Legendre series and show that only the magnetic dipole remains. Note that the absorption cross section is exactly the opposite to the scattering cross section, i.e., how much of the incoming radiation gets absorbed. It is defined as:

$$\sigma_{abs} = \frac{\text{Powerloss}}{\text{Incident energy flux}}.$$

3. (*30 points*) A weather radar wave is scattered by a raindrop of radius a such that $ka << 1$, i.e., the long wavelength limit applies. The raindrop has a real dielectric constant ϵ and a small real conductance σ such that the skin depth δ is large compared to the radius a. The incident wave is plane polarized in the vertical direction.

   (a) Calculate the differential scattering cross section.

   (b) Calculate the total scattering cross section.

   (c) Calculate the total absorption cross section, defined as the ratio of absorbed power over incident power flux. The former is given by the Poynting flux into the raindrop, but be careful to use the total fields in that calculation, i.e., the sum of incident and scattered fields. This is a somewhat lengthy calculation with up to quadruple cross products. However, clever application of Poynting's theorem can make this calculation much easier!

4. (*10 points*) Show explicitly that two consecutive Lorentz transformations in the same direction with velocity v_1 and v_2, respectively, are equivalent to a single transformation with $v = (v_1 + v_2)/(1 + v_1 v_2/c^2)$.

5. (*10 points*) A plane monochromatic electromagnetic wave propagating in free space is incident normal to a plane mirror surface where it is reflected. Obtain the frequency of the reflected wave in the case that the mirror moves at speed v, not necessarily small compared to c, with respect to the observer. Assume that $\mathbf{v}$, $\mathbf{k}$, and $\mathbf{n}$ (the normal to the mirror surface) are all parallel.

6. (*40 points*) Seeing versus observing:

   (a) In the following, assume β=0.9. A rod of length L'=10 m is aligned with, and moving along the x-axis towards you at speed v. You, the observer, are located slightly above the x-axis so you can see both ends of the rod (Note: *seeing* means that a photon arrives at your eye.)

   (i) What is the rod's length L that you observe? How would you measure it?

   (ii) What is the rod's length L_s^+ that you *see*? Why is it different from (i)? (The effect is called *abberation*).

   (iii) What length L_s^- would you *see* if the rod moves away from you at speed v?

   (iv) How would (ii) and (ii) turn out if Galilean relativity applied?

   (b) A star appears to move across the sky at a speed larger than c (assume you know the distance of the star and its apparent angular motion, but not it its radial motion). Such observations sometimes lead to sensational reports that Einstein's physics is violated. How can this happen? Discuss how the apparent speed depends on the angle between the star's velocity and the line of sight. At which angle does the apparent speed maximize? What would be different if Galilean relativity applied?