

## PHYS 942 homework assignment #02

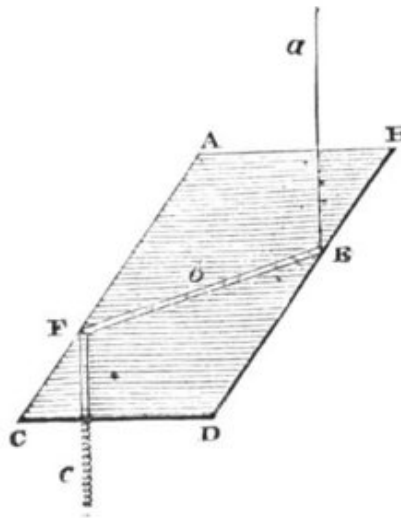
Department of Physics  
University of New Hampshire  
Prof. J. Raeder, J.Raeder@unh.edu

PHYS 942  
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Names ( $\leq 3$ , write clearly): \_\_\_\_\_

Due: Friday, September 23, at the lecture. **Show all your steps!**

- (10 points) Derive equation 7.43 in Jackson.
- (30 points) The figure below shows a *Fresnel rhombus*. Incident light coming from  $a$  is internally reflected twice and exits at  $c$ . For a suitable choice of parameters, a Fresnel rhombus converts linearly polarized light into circularly polarized light. Hint: Jackson mentions the *Fresnel rhombus* on page 308.



- Explain how the Fresnel rhombus works.
  - Determine the orientation of the polarization vector of the incoming light and the angle of incidence at  $E$  that is required to convert plane polarized light into circularly polarized light if the index of refraction of the glass is  $n=1.51$ .
- (40 points) Jackson, problem 7.3. Make sure you include all transmitted and reflected wave fields. The transmission coefficient is the ratio of the square amplitudes of the exiting to the incident wave. The effect studied here is the basis of a *Fabry-Perot* interferometer.

4. (20 points) Consider an isothermal atmosphere with temperature  $T$  in equilibrium.

- (a) Using the momentum equation for hydrostatics  $\nabla p = n\mathbf{g}$ , where  $\mathbf{g}$  is the gravitational acceleration and  $p$  the atmospheric pressure, calculate the density  $n(z)$  as a function of altitude  $z$  (positive upward) if the density on the ground is  $n_0$ .
- (b) Now assume radiation is entering the atmosphere from above at an angle  $\vartheta$  to the zenith and at radiation density  $J_\infty$ . Assuming the cross section of absorption is  $\sigma$  show that the intensity of the radiation obeys:

$$dJ(z) = n(z)\sigma \frac{dz}{\cos \vartheta} J(z)$$

whereas the absorption rate per unit volume is  $q(z) = \cos \vartheta (dJ/dz)$ . Now, show that

$$q(z) = q_0 \exp\left(1 - y - \frac{1}{\cos \vartheta} \exp(-y)\right)$$

where  $y = (z - z_0)/H$ ,  $H$  is the scale height from part (a),  $q_0$  is the maximum of  $q(z)$  over  $z$ , and  $z_0$  is the height at which the maximum occurs, i.e.,  $q_0 = q(z_0)$ .

- (c) Make plots of  $q(z)/q_0$  for  $\vartheta = 0^\circ$ ,  $45^\circ$ , and  $85^\circ$ . You should see that the absorption rate (in case of the ionosphere at which electrons are produced, but this formalism is also valid for other photo-reactions such as the ozone layer) forms a relatively thin layer. Such a layer is called a *Chapman layer*.