Lec24 IAM550 J. Raeder 11/21/2019 Differential equations, Linear systems

Announcements:

- Final: in-class final, 2h → one as originally scheduled, Tuesday 12/17 10:30 – 12:30, the other one in the week 12/10-13, TBD.
- Remaining schedule:
    - lecture today
    - lecture Tuesday 12/3 **review**
    - labs week after THX December 3,5
    - no more homework

Systems of linear equations: another example

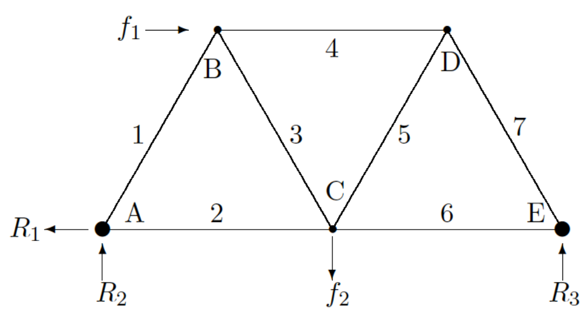

Figure 9.1: An equilateral truss. Joints or nodes are labeled alphabetically, A, B, . . . and Members (edges) are labeled numerically: 1, 2, ... The forces f_1 and f_2 are applied loads and R_1, R_2 and R_3 are reaction forces applied by the supports.

Gives rise to the following system of linear equations:

$$
\begin{aligned}
.5\,T_1 + T_2 &= R_1 = f_1 \\
.866\,T_1 &= -R_2 = -.433\,f_1 - .5\,f_2 \\
-.5\,T_1 + .5\,T_3 + T_4 &= -f_1 \\
.866\,T_1 + .866\,T_3 &= 0 \\
-T_2 - .5\,T_3 + .5\,T_5 + T_6 &= 0 \\
.866\,T_3 + .866\,T_5 &= f_2 \\
-T_4 - .5\,T_5 + .5\,T_7 &= 0,
\end{aligned}
$$

Or in matrix form:

$$
\left(\begin{array}{ccccccc|c}
.5 & 1 & 0 & 0 & 0 & 0 & 0 & f_1 \\
.866 & 0 & 0 & 0 & 0 & 0 & 0 & -.433 f_1 - .5\, f_2 \\
-.5 & 0 & .5 & 1 & 0 & 0 & 0 & -f_1 \\
.866 & 0 & .866 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -.5 & 0 & .5 & 1 & 0 & 0 \\
0 & 0 & .866 & 0 & .866 & 0 & 0 & f_2 \\
0 & 0 & 0 & -1 & -.5 & 0 & .5 & 0
\end{array}\right).
$$

Obviously, this would not be easy to solve by hand.

Solution procedure: Gauss Elimination

Basic rules: The following does not change the solution:

1. Multiplying any equation with a constant
2. Adding 2 equations
3. Swapping equations

Basic idea:

1. Elimination: make the lower triangle below the diagonal zero by multiplying and adding. Column by column downward, starting from first column.
2. Back substitution: Now solve for equations from bottom to top.

Example. Do on the board, but only w/matrix. Also, avoid fractions.

System of equations	Row operations	Augmented matrix
$\begin{aligned} 2x + y - z &= 8 \\ -3x - y + 2z &= -11 \\ -2x + y + 2z &= -3 \end{aligned}$		$\left[\begin{array}{ccc\|c} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{array}\right]$
$\begin{aligned} 2x + y - z &= 8 \\ \tfrac{1}{2}y + \tfrac{1}{2}z &= 1 \\ 2y + z &= 5 \end{aligned}$	$L_2 + \tfrac{3}{2}L_1 \to L_2$ $L_3 + L_1 \to L_3$	$\left[\begin{array}{ccc\|c} 2 & 1 & -1 & 8 \\ 0 & \tfrac{1}{2} & \tfrac{1}{2} & 1 \\ 0 & 2 & 1 & 5 \end{array}\right]$
$\begin{aligned} 2x + y - z &= 8 \\ \tfrac{1}{2}y + \tfrac{1}{2}z &= 1 \\ -z &= 1 \end{aligned}$	$L_3 + -4L_2 \to L_3$	$\left[\begin{array}{ccc\|c} 2 & 1 & -1 & 8 \\ 0 & \tfrac{1}{2} & \tfrac{1}{2} & 1 \\ 0 & 0 & -1 & 1 \end{array}\right]$
The matrix is now in echelon form (also called triangular form)		
$\begin{aligned} 2x + y &= 7 \\ \tfrac{1}{2}y &= \tfrac{3}{2} \\ -z &= 1 \end{aligned}$	$L_2 + \tfrac{1}{2}L_3 \to L_2$ $L_1 - L_3 \to L_1$	$\left[\begin{array}{ccc\|c} 2 & 1 & 0 & 7 \\ 0 & \tfrac{1}{2} & 0 & \tfrac{3}{2} \\ 0 & 0 & -1 & 1 \end{array}\right]$
$\begin{aligned} 2x + y &= 7 \\ y &= 3 \\ z &= -1 \end{aligned}$	$2L_2 \to L_2$ $-L_3 \to L_3$	$\left[\begin{array}{ccc\|c} 2 & 1 & 0 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$
$\begin{aligned} x &= 2 \\ y &= 3 \\ z &= -1 \end{aligned}$	$L_1 - L_2 \to L_1$ $\tfrac{1}{2}L_1 \to L_1$	$\left[\begin{array}{ccc\|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$

Code: Live!