Lec22 IAM550 J. Raeder 11/14/2019 Differential equations

Announcements:

- Midterm II grades by the end of the week.
- Vote on final:
  - In class, Tuesday 12/17 10:30 – 12:30
  - Take-home, Tuesday 12/17 9am – 5pm
  - Take-home, Saturday 12/14 9am – 5pm (are buildings accessible?)

All on the blackboard or real time MATLAB

Recap: ODE, Euler forward method

Demonstrate: Euler forward instability

Better methods:

- Euler backward → stable, but implicit
- Predictor - Corrector → similar stability, but more accurate, show graphically

$$y' = f(t, y), \quad y(t_0) = y_0$$

do one Euler forward step:

$$\tilde{y}_{i+1} = y_i + hf(t_i, y_i)$$

and average with an Euler backward step, using the predicted values:

$$y_{i+1} = y_i + \tfrac{1}{2}h\big(f(t_i, y_i) + f(t_{i+1}, \tilde{y}_{i+1})\big).$$

Alternatively, make a half forward step, then evaluate slope at predicted point (second order Runge-Kutta):

$$y_{n+1} = y_n + hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hf(t_n,\ y_n)\right).$$

→ explain on bb.

Most popular: 4th order **Runge-Kutta:**

$$
\begin{aligned}
k_1 &= h\, f(t_n, y_n), \\
k_2 &= h\, f\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right), \\
k_3 &= h\, f\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right), \\
k_4 &= h\, f\left(t_n + h, y_n + k_3\right).
\end{aligned}
$$

$$
\begin{aligned}
y_{n+1} &= y_n + \tfrac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right), \\
t_{n+1} &= t_n + h
\end{aligned}
$$

→ very efficient and accurate, easy to program, only needs 4 function evaluations for every step.

Back to pendulum equation, the non-linear version:

$$\frac{d^2\Theta}{dt^2} = -\frac{g}{L}\sin(\Theta)$$

Rewrite as a system of two first order equations:

$$\frac{d\Theta}{dt} = V$$
$$\frac{dV}{dt} = -\frac{g}{L}\sin(\Theta)$$

or in vector form, the way we will do it in MATLAB:

$$\frac{d}{dt}\begin{bmatrix}\Theta \\ V\end{bmatrix} = \begin{bmatrix}V \\ -\sin(\Theta)\end{bmatrix}$$

Note: the RHS is what we called

$$f(t_n, y_n)$$

Here it does not depend on t.

→ write code for Euler, Predictor-Corrector, and RK4
→ use function for RHS
→ plot comparisons
→ add solution for small amplitude