Lec17 IAM550 J. Raeder 10/28/2019 while loops, functions

Announcement:

- Midterm I has been graded, returned, and grades entered into canvas
- Midterm II scheduled for Thursday, November 7, during class time, 60 minutes.

All on the blackboard or real time MATLAB

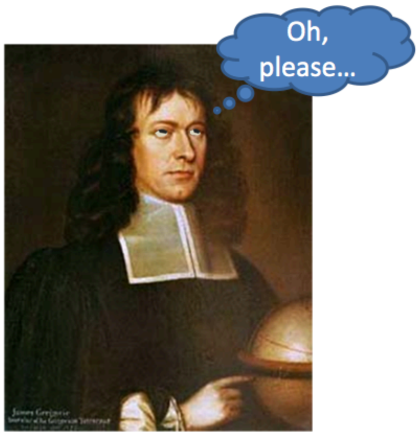
- Functions:
  - In separate m-files: we did that, but from now on keep all in one file for simplicity.
  - mylog(x) use while to return for given precision (for -1 < x < 1):
  - $$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} x^k = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots,$$
  - functions with more than 2 return values → fit function.
  - Global versus local variables, again.

Introduction to Taylor series

The Taylor theorem: Any smooth function can be approximated as a polynomial.

The Taylor series: A practical way to mathematically express and use the Taylor theorem.

Brook Taylor, 1685-1731



James Gregory, 1638-1675

Colin Mclaurin, 1698-1746

The Taylor Series

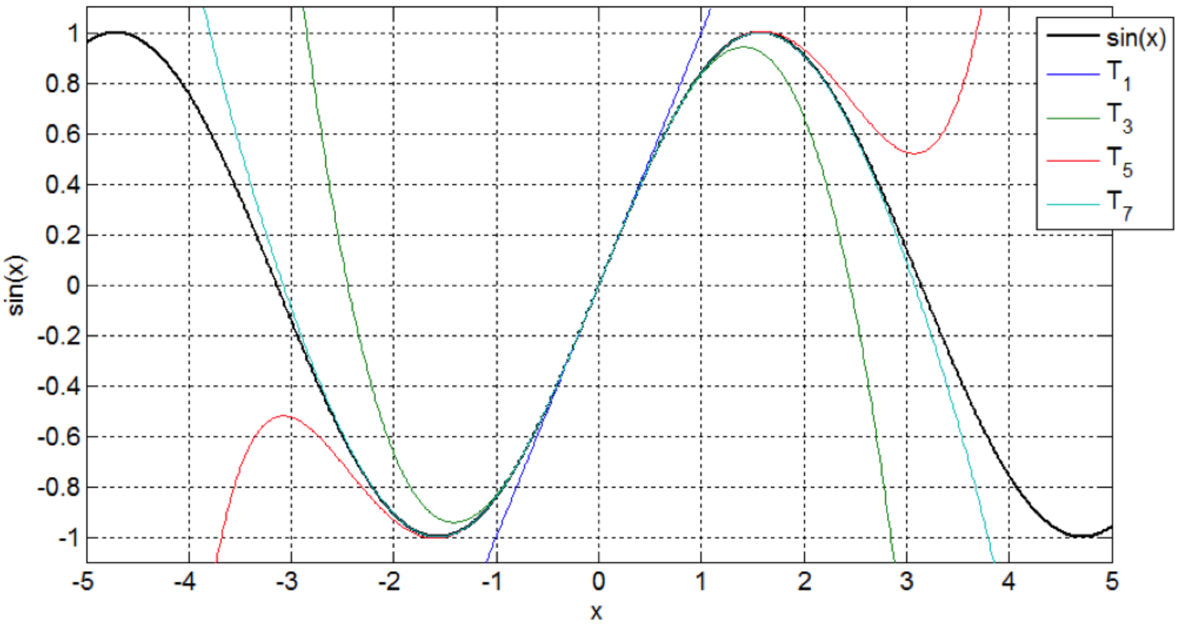
Allows us to predict the value of a function at t_{i+1} based on knowledge of the function at t_i.

step size: $dt = t_{i+1} - t_i$

$$f(t_{i+1}) \approx f(t_i) + f'(t_i)dt + \frac{f''(t_i)}{2!}dt^2 + \frac{f^{(3)}(t_i)}{3!}dt^3 + \cdots + \frac{f^{(n)}(t_i)}{n!}dt^n$$

n^{th} order approximation

$$sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$



remainder term: $$R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!} dt^{n+1}$$

A complete Taylor series expansion looks like this:

$$f(t_{i+1}) = f(t_i) + f'(t_i)dt + \frac{f''(t_i)}{2!}dt^2 + \frac{f^{(3)}(t_i)}{3!}dt^3 + \cdots + \frac{f^{(n)}(t_i)}{n!}dt^n + R_n$$

Note the ‘equal’ sign and the addition of a **‘remainder term’**

$$R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}dt^{n+1}$$

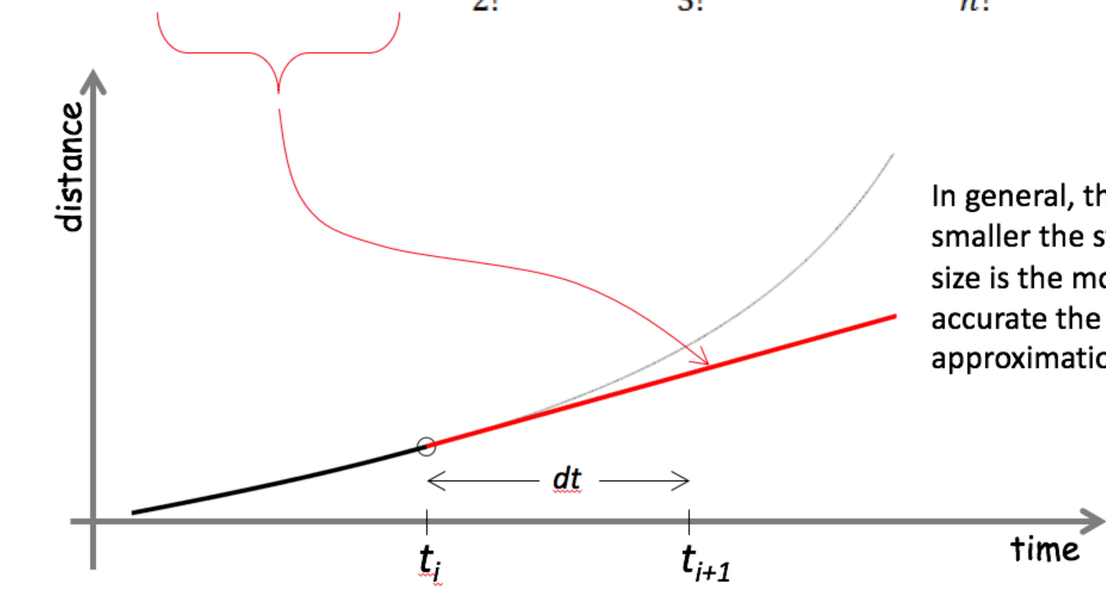
$$t_i \leq \xi \leq t_{i+1}$$

If we knew the remainder term, we wouldn’t be making an approximation (we’d know the exact answer!).

The Taylor Series: step size

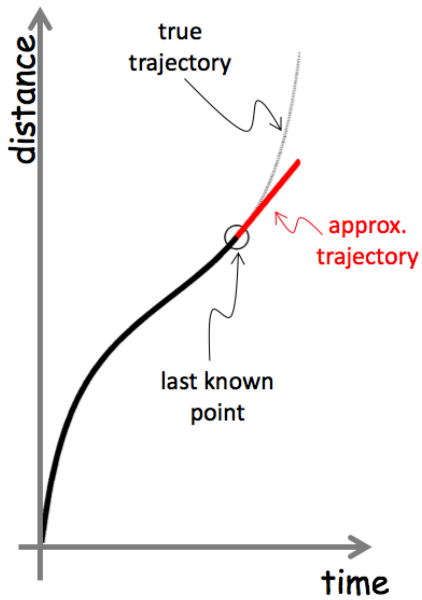
step size: $dt = t_{i+1} - t_i$

$$f(t_{i+1}) \approx f(t_i) + f'(t_i)dt + \frac{f''(t_i)}{2!}dt^2 + \frac{f^{(3)}(t_i)}{3!}dt^3 + \cdots + \frac{f^{(n)}(t_i)}{n!}dt^n$$



In general, the smaller the step size is the more accurate the approximation.

Numerical Differentiation



Truncated 1st Order Taylor Series:

$$f(t_{i+1}) = f(t_i) + f'(t_i)dt + R_1$$

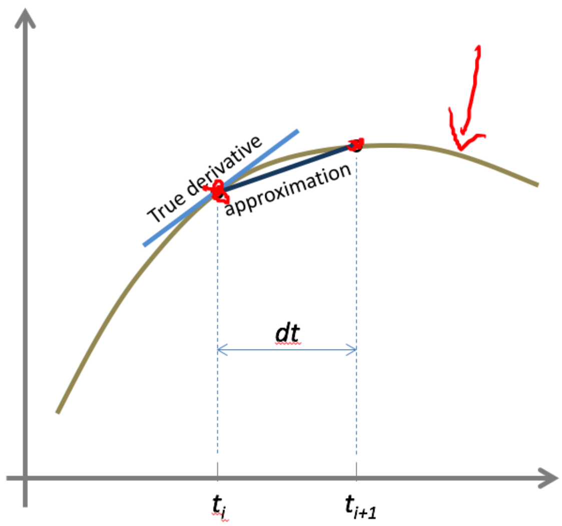
What if we were interested in estimating the velocity?

$$f'(t_i) = \frac{f(t_{i+1}) - f(t_i)}{dt} - \frac{R_1}{dt}$$

$$\frac{R_1}{dt} = \frac{\frac{f^{(2)}(\xi)}{2!}dt^2}{dt} = \frac{f^{(2)}(\xi)}{2!}dt = O(dt)$$

We don't know what the error in our velocity estimate is, but we can say it is of the *order* of the step size.

Numerical Differentiation



$$f'(t_i) \approx \frac{f(t_{i+1}) - f(t_i)}{dt}$$

This approximation for the first derivative of *f* is called a 'forward difference'.

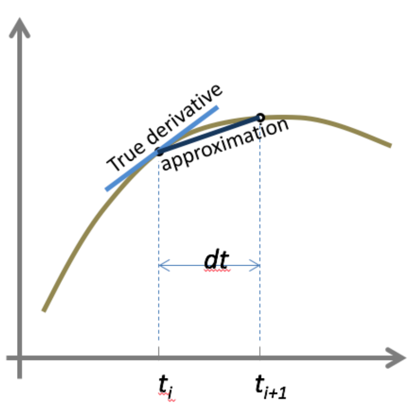
Derivatives are ubiquitous in science and engineering!

Newton's 2nd Law: $F = ma$

Newton's 2nd Law as expressed by Newton: $F = \frac{d(mv)}{dt}$

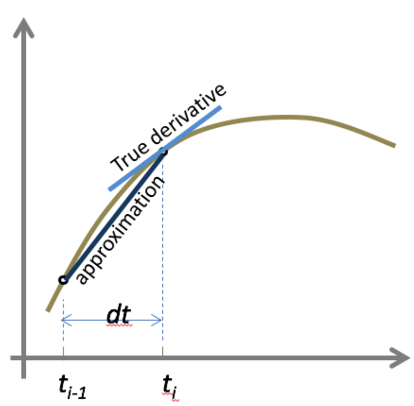
forward difference

$$f'(t_i) \approx \frac{f(t_{i+1}) - f(t_i)}{dt}$$



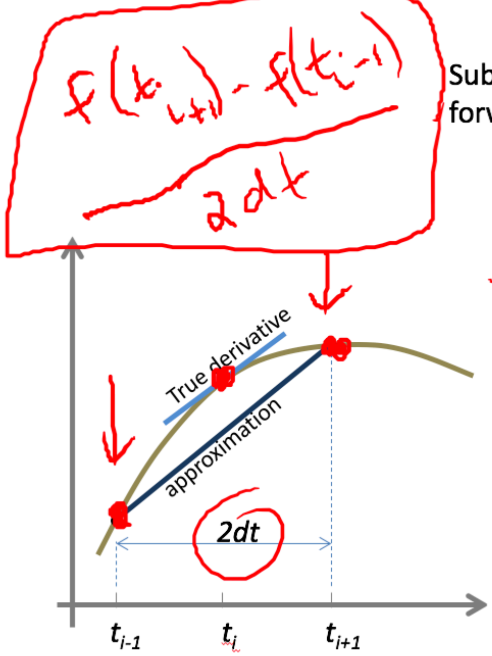
backward difference

$$f'(t_i) \approx \frac{f(t_i) - f(t_{i-1})}{dt}$$



Both estimates are *O*(*dt*)

A centered finite-difference seems like it would be more accurate....



Subtract the backward Taylor expansion from the forward Taylor expansion:

$$f(t_{i+1}) = f(t_i) + f'(t_i)dt + \frac{f''(t_i)dt^2}{2!} + \cdots$$

$$f(t_{i-1}) = f(t_i) - f'(t_i)dt + \frac{f''(t_i)dt^2}{2!} - \cdots$$

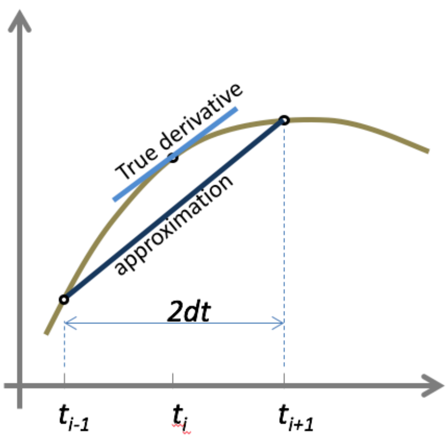
$$f(t_{i+1}) - f(t_{i-1}) = 2f'(t_i)dt + 2\frac{f^{(3)}(t_i)dt^3}{3!} + \cdots$$

... solve for $f'(t_i)$

$$f'(t_i) = \frac{f(t_{i+1}) - f(t_{i-1})}{2dt} - \frac{f^{(3)}(t_i)dt^2}{3!} + \cdots$$

A centered finite-difference seems like it would be more accurate….

$$f'(t_i) = \frac{f(t_{i+1}) - f(t_{i-1})}{2dt} - \frac{f^{(3)}(t_i)dt^2}{3!} + \cdots$$



This can be calculated if we already know *f* – perhaps its our data, or perhaps we have a model for it. (note that this is a non-causal representation of our data if t_i is now!).

Presumably we don't know this 3[rd] derivative since we're only just now just figuring out the 1[st] derivative!

A key component in examining this solution is realizing that this last (unknown) term is O(dt^2) .

This is a more accurate representation of the derivative. If we halve *dt*, we quarter the truncation error with the centered difference wheras we'd halve the truncation error with a forward or backward difference.

Take-home messages

- We've extended our ideas about Taylor series to first and second derivatives: Forward, backward, centered
- We also examined the remainder term so that we could estimate the order of accuracy for these finite differences
- So much for now about differentiation and finite differences. We will come back to that when we try to solve simple ordinary differential equations

Now to HW 03: What generation do you belong to?

- Silent (great?) generation
- Boomers
- Gen X
- Gen Y, Millenials
- Gen Z
- What next?

Or maybe:

NPR › 2012/09/03 › are-todays-millennials-the-screwed-generation

Are Today's Millennials The 'Screwed Generation'? : NPR

Sep 3, 2012 - Are Today's Millennials The '**Screwed Generation**'? ... Money Newsletter. Just the right amount of economics, sent **weekly**. E-mail address ...

Forbes › sites › victorlipman › 2012/07/26 › has-a-privileged-generati... ▾

Has A Privileged Generation Become The Screwed Generation?

Jul 26, 2012 - A recent article in Newsweek is a dark but thought-provoking piece. Face it, as a **generation** we Boomers haven't exactly been great stewards ...

Missing: ~~week~~ | Must include: week

Newsweek › are-millennials-screwed-generation-65523 ▾

Are Millennials the Screwed Generation? - Newsweek

Jul 16, 2012 - The **screwed generation** also enters adulthood loaded down by a mountain of ... "I've been applying to five jobs a **week** and have gotten nothing but rejections." ... with college graduates for basic jobs in service **businesses**.

Business Insider › how-chart-shows-how-millenials-got-screwed-2017... ▾

One chart shows how millennials got screwed - Business Insider

Nov 16, 2017 - In fact, Britain's so-called millennial **generation** — those born from ... when he announces next **week's** Autumn Budget, including a freeze on ...

What can you do about that?

Better invest smart! →

- Compound interest.
- Continuing investing (Dollar Cost Averaging, DCA, although that term is often used in a different context). Surprisingly, market crashes are actually good for you! Not so much when you are about to retire, though.
- Test this with historical stock market data (SP500 back to 1927) → backtesting.
- Use random variables to estimate risk.
- Along the way: some more fit (exponential fit), histograms.