IAM 550 Introduction to Engineering Computing
Computer Lab 6
Newton-Raphson root finding, functions, formatting
J. Raeder, October 1/3

Objectives:

This week you will be finding the roots of Legendre functions. In doing so, you will need to

- Use the Newton-Raphson method to find the roots of an equation.
- Use functions and function handles.
- Know how to format tables

Deliverables **due no later than 2 days after the end of your lab session:**

A MATLAB diary for your laboratory session (25% of your laboratory grade). This should be submitted via blackboard as an assignment no later than 2 days after your lab.

Deliverables **due at the beginning of your next lab session (October 15 or 17):**

A lab report summarizing your results and answering any questions asked in the lab instructions, and including any MATLAB files that have been requested. Make sure your name is on *all* pages of your lab report (also the m-files, as a comment line).

Background:

The Legendre polynomials (L.P.) play an important role in the solution of spherical problems and for Gauss-Legendre numerical integration. In particular for the latter, one needs the roots of the L.P.. Since they are polynomials, they are defined for all real numbers, but they are usually only of interest on the interval [-1,1]. More specifically, the values are bound by [-1,1] on that interval, the sequence of polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$, … forms an Ortho Normal Set (ONS), and each L.P. $P_n(x)$ of order n has exactly n roots in the interval [-1,1]. There are numerous ways to define the L.P., but the easiest is the recursion (see https://en.wikipedia.org/wiki/Legendre_polynomials for details):

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)\,.$$

with:

$$P_0(x) = 1\,, \quad P_1(x) = x.$$

A similar recursion formula exists for the derivatives:

$$\frac{d}{dx}P_{n+1}(x) = (n+1)P_n(x) + x\frac{d}{dx}P_n(x)\,.$$

Task 1 of 2

Write a MATLAB external function (m-file) that calculates the L.P. values, given the order n and argument x, and returns both $P_n(x)$ and the derivative of $P_n(x)$. Test the function by writing a script that plots $P_n(x)$ for n=1,…,12 on the interval [-1,1] in one figure. This should give 2 m-files and one plot.

Task 2 of 2 (optional, for an additional 50 points)

Write a script that uses the Newton-Raphson method to automatically find all zeros of $P_n(x)$ on the interval [-1,1] for n=12. You need to 'march' systematically through the interval to find the candidates, and then refine the starting guesses with Newton-Raphson. The N-R root finding should be in a separate m-file, and you should use file handles to pass the function for the L.P. and its derivative. The output should be a nice table with n versus root, printed using fprintf() so that it can be copied and pasted as text into the report appendix. The format should reflect the actual precision of the numbers. You need to find a mono-spaced font to make sure the numbers line up properly in the report.

Turn in all m-files, the table, and the plot. Make sure the plot is nicely labelled.