IAM 550
Midterm Exam II
November 7, 2019

No books. No cheat sheets. No cell phones. No computers.
Only pen and paper. You should not need extra paper (but there is some, in case).
Make sure your answers can be understood. Write your name on every page as indicated.

Printed Student Name: SOLUTION

Grading: pts / 100 pts (Grader:)

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1. [10 pts] Name at least 3 algorithms to calculate the integral of a function, ordered from least accurate to most accurate.

    a. First: Riemann sum

    b. Second: trapezoidal rule

    c. Third: Simpson 1/3 rule

2. [20 pts] Write a WHILE loop that does exactly the same as this FOR loop:

```
for i=1:2:13
    A(i)=cos(pi*i)
end
```

```
i=1;
while i <= 13
   A(i)=cos(pi*i);
   i=i+2;
end
```

Name:

3. [30 pts] Write a function named TRAPEZ in MATLAB that takes vectors X and Y of equal length and returns the integral, using the trapezoidal rule. The points of X are NOT evenly spaced.

```
function r=TRAPEZ(X,Y)
r=0;
n=length(X)-1;
for i=1:n
    r=r+0.5*(X(i+1)-X(i))*(Y(i)+Y(i+1));
end
end
```

4. [40 pts] Suppose you are the worst dart thrower in the world. Instead of focusing your throws on a spot, your throws are perfectly even distributed over the board. Suppose the board is 1 m by 1 m, and you draw a circle of radius 0.5 m that fits exactly on the board. You now throw a large number of darts, with 'NB' darts hitting the board (assume NB=1000) and 'NC' darts landing within the circle.

   a. Explain how you can use the ratio of the counts to calculate an approximation of π. Hint: consider what the counts are proportional to. Drawing a picture may also help.

      The ratio of the hits is roughly the same as the ratio of the areas, thus
      NC/NB ~ mypi*0.5^2 / 1= mypi/4 → mypi ~ 4*(NC/NB)

   b. Write a MATLAB script that uses random numbers to simulate throwing 'NB' darts that land on the board, that is in the area [0,1]x[0,1], counts the number of hits 'NC' within the circle, and computes the estimate 'mypi'. Add code that calculates the true error 'TE', the relative error 'RE', and the percentage error 'PE' of your calculation. Note that

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the variable 'pi' is predefined in MATLAB with the true value of π. Note: even if you could not solve (a), you can still write most of this code.

```
NB=1000;   %  throws on the board
NC=0;      %  hit counter
for i=1:NB
    x=rand;
    y=rand;   % random throw
    r=sqrt( (x-0.5)^2 + (y-0.5)^2 ); %  distance from center
    if r < 0.5  %  within the circle
        NC=NC+1;  %   count the hit
    end
end

mypi=4*(NC/NB)

TE=pi-mypi
RE=TE/pi
PE=100*RE
```